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A TREATISE

ON THE

STRENGTH, FLEXURE, AND STIFFNESS

OF

CAST IRON BEAMS AND COLUMNS,

ETC. ETC.

LONDON:
J. MOYES, CASTLE STREET, LEICESTER SQUARE.

A TREATISE

ON THE

STRENGTH, FLEXURE, AND STIFFNESS

OF

CAST IRON BEAMS AND COLUMNS,

SHEWING THEIR FITNESS TO RESIST

TRANSVERSE STRAINS, TORSION, COMPRESSION, TENSION, AND IMPULSION;

WITH

TABLES OF CONSTANTS,

TO BE USED FOR CALCULATING THE STRENGTH, FLEXURE, AND STIFFNESS OF SIMILAR BEAMS AND COLUMNS OF WROUGHT IRON, AND SEVERAL SORTS OF TIMBER GENERALLY EMPLOYED FOR ARCHITECTURAL AND MECHANICAL PURPOSES:

TO WHICH IS ADDED,

BY WAY OF APPENDIX,

A COLLECTION OF RULES IN WORDS AT LENGTH,

FOR CALCULATING THE MOST IMPORTANT PRACTICAL CASES INVESTIGATED IN THE COURSE OF THE WORK, BEING INTENDED AS A GUIDE FOR THOSE WHO ARE NOT VERSED IN ALGEBRAIC REDUCTIONS.

BY WILLIAM TURNBULL.

LONDON:
JOSIAH TAYLOR, ARCHITECTURAL LIBRARY,
59, HIGH HOLBORN.

1832.

PREFACE.

In laying this little work before the public, the Author has to observe, that the plan of it was formed during the period of his employment with Messrs. Cottam and Hallen, Winsley Street, Oxford Street, London, where he had frequent occasion to direct his attention to the mode of calculating the strength and deflexion of beams and columns, of various forms and dimensions, cast by them for bearers and supports in buildings and other extensive works. To further his object in this respect, Mr. Tredgold's valuable Essay on the Strength of Cast Iron was put into his hand; but not having previously consulted the writings of this eminent author, the aid thus afforded was insufficient for the purpose. Mr. Tredgold enters largely into the detail and comparison of various practical experiments; but from a perusal of these details the writer of this work did not derive the information he wished, the

immediate object of his research being to determine by calculation the strength and deflexion of beams that were to be experimented upon, previous to being applied to their intended purpose.

The difficulty of obtaining suitable formulæ, or modes of calculation for any particular case, induced the Author to peruse the above-named work with some attention, and from this perusal the plan of his own performance was suggested. It occurred to him that the theory, as delivered in Mr. TREDGOLD's work, might be more simply developed, and the formulæ arising therefrom, collected and arranged in such a manner as to enable practical persons to refer at once to an expression applicable to the very case proposed: this is the chief object to which the writer's attention has been directed, and he flatters himself that he has in some measure succeeded.

The mode of comparison here employed for establishing the theory, cannot, it is presumed, from its simple and obvious nature, be easily misunderstood; the principle is not new, but it does not appear to have hitherto been adopted as the ground-work of a similar

doctrine: the theory of *strength*, of *deflexion* and *stiffness*, as delivered in the following pages, depends entirely upon it; and by using the same experimental results as the author above-named, we have in every instance arrived at the same conclusions. This is so far satisfactory; for, admitting the material on which the experiment was tried to be perfectly free from defects, and uniform in every part, the formulæ deduced from the experimental numbers may be applied in other cases, where the material is the same, with mathematical certainty; and even in cases where the material is different, the results will be sufficiently accurate to apprise the engineer of its fitness or unfitness for the intended purpose.

The Author thinks it not amiss to remark that, during the progress of the work he has consulted no other writer on the strength of materials but Mr. Tredgold, and, consequently, according as the principles of his prototype are true or false, the theory here sought to be established must stand or fall. The manuscript was submitted to the inspection of Thomas Telford, Esq., President of the Institution of Civil Engineers; and, but for the

encouragement which he held out, aided by the liberality of the publisher, the attempt would have been entirely abandoned.

Having said this much respecting the plan and circumstances attending his work, the Author submits it to the public eye, with a sincere hope that his labours will be useful to those persons inquiring on the subject; and should the reception of the work be such as to occasion the demand for another edition, it may experience further improvements.

29, *Hertford Street, Fitzroy Square.*
November 1831.

ERRATA.

PAGE
21—11th line from bottom, *for* 1885, *read* 942; and *for* 8915, *read* 9858.
23—4th line from top, *for* 4, *read* 6.
7th line from bottom, *for* 1080, *read* 1620.
5th line from bottom, *for* 1080, *read* 810; and *for* 80082, *read* 80352.
118—6th line from top, *for* $\sqrt{6{\cdot}208}=2{\cdot}49$, *read* $\sqrt{6{\cdot}3947}=2{\cdot}52$.
133—13th line from top, *for* 7, *read* ·7.
167—13th line from top, *for* fourth part, *read* fourth power.

A

TREATISE

ON

CAST IRON BEAMS,

WHEN EXPOSED TO A TRANSVERSE STRAIN.

THE doctrine of the strength of materials originated with the illustrious Galileo, and since his time many eminent mathematicians and philosophers have laboured to perfect the theory and to reduce its principles to numerical calculation; but their endeavours have chiefly been directed to the discovery of rules for calculating the ultimate strength, or the force excited in the material at the instant of fracture, and hence have determined the dimensions of beams that will just break with a given strain: to those, however, who consider the subject with a little attention, it will soon appear, that this limit of strength ought never to be reached in practice, for beams should not be exposed to a strain capable of producing this effect.

It has been found, that when a material is loaded beyond a certain point, its elastic force is impaired, and it loses the power of restoring itself to its

natural state when the load is removed; and Dr. Young has justly remarked, that when this is the case, the value of the material is destroyed for bearing purposes, because a small addition to the straining force is sufficient to increase the deflexion or bending of the beam till fracture takes place: hence it appears, that rules designed to assist the mechanic in the proper construction of beams, should be so framed as to give results within the elastic power of the material, for in that case, the load to which they are exposed will be sustained with safety.

The subject chosen is cast iron, a material almost universally employed in works where strength and durability are objects of consideration; the strain on which we have treated is transverse; the data are derived from experiment, and, consequently, the rules which we are about to establish must partake of the accuracy or inaccuracy of the results from which they are derived.

The forms of beams most commonly exposed to transverse strains in practical cases are, the *rectangular*, the *square*, the *cylindrical*, the *tubular*, the *grooved*, or that which has its transverse section in form of the letter I, and the *open*,* or that whose

* This is a particular case of the rectangular beam, its longitudinal section is

middle part is entirely left out, with the exception of cross stays, to prevent the upper and lower parts from coming together: each of these forms or varieties admits of six cases, distinguished by the position of the load and the manner of fixing or supporting the beam; and these are as below:

1. *Supported at the ends, and loaded in the middle.*

1.

2. *Supported at the ends, but the load not in the middle.*

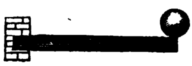
1/4

3. *Supported at the ends, and loaded uniformly over the length.*

2.

4. *Fixed at one end, and loaded at the other.*

1/4



5. *Fixed at one end, and loaded uniformly over the length.*

1/2

6. *Supported at the ends, and the load increasing as the distance from one of the supports.*

3/

1/3

In deriving expressions to involve the strength and dimensions of the beams in these several cases, a constant number or co-efficient will arise for each, having its value dependent on the cohesive force and extensibility of the material employed, and these constants will apply to similar beams of the same material, whatever may be the dimensions. Now, the writers on the resistance of solids have shewn, that if the co-efficient for a beam supported and loaded as in the first case be unity, those for beams supported and loaded as in the other cases will be respectively $\frac{1}{4}$, 2, $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{31}{16}$, (*see* TREDGOLD on Cast Iron, art. 106, 111, 114, 116, 118, and 122); therefore, whatever constant is derived from experiment for the first case, the others must bear such proportion to it as the above numbers do to unity.

This being premised, we shall proceed with the solution of the following problems:

PROBLEM I. *To determine an expression for the dimensions of a uniform rectangular beam of cast-iron, to bear a given load; the length of the beam being given, and the elastic force of the material remaining perfect.*

It is proved by the writers on mechanics, that the lateral strength of a rectangular beam is directly

as its breadth and square of the depth, and inversely as its length (*see* TREDGOLD on Cast Iron, art. 79); hence we infer, *that the breadth drawn into the square of the depth, divided by the length drawn into the load, is a constant quantity;* therefore, if the strength of a beam, whose dimensions are known, be accurately determined by experiment, the strength of beams of the same material, and of any other dimensions, can be easily found.

For, let B, D, L and W, be the breadth, depth, length and weight used in the experiment, and b, d, l and w, those for which the calculation is made, then, from what has been stated above, we have $B\,D^2 \div L\,W = b\,d^2 \div l\,w$, which, by reducing the fractions, becomes $l\,w\,B\,D^2 = L\,W\,b\,d^2$. Now, it has been found, by very accurate experiments, that a bar of soft gray cast iron, 1 inch square and 34 inches between the supports, will bear a load of 300 lbs. in the middle of its length, while the elastic force remains perfect (*see* TREDGOLD on Cast Iron, art. 45); therefore, by substituting these numbers in the preceding equation, and taking the length in feet, we get $12\,l\,w = 10200\,b\,d^2$, and dividing both sides of this equation by 12, we obtain

1. *When the beam is supported at the ends, and loaded in the middle,*

$$l\,w = 850\,b\,d^2.$$

2. *When the beam is supported at the ends, but the load not in the middle.* Let m and n be the segments into which the length of the beam is divided by the direction of the straining force; then, because the strain at the point where the load is applied, is proportional to the rectangle of the segments into which the length of the beam is divided at that point (*see* TREDGOLD on Cast Iron, art. 100, *or* GREGORY's Mechanics, vol. i. art. 178, cor. 2),* it is, when compared with the strain in the last case,

$$m\,n\,w = 212\tfrac{1}{2}\,l\,b\,d^2.$$

3. *When the beam is supported at the ends, and loaded uniformly over the length.* It is a known fact in mechanics, that the strain produced by a load uniformly diffused throughout the length of a beam,

* This is an important property in the construction of cast iron beams: from it we learn, that the cross section may be diminished from the point of greatest strain towards the supported points, in a certain ratio, without any diminution of useful strength; and hence, a great saving of the material can be effected, or a considerable accession of strength obtained with the same quantity of material; but recollect that the stiffness of the beam is diminished when moulded after this manner. The breadth of the beam remaining constant, the outline of the depth, when formed according to the analogy, is an ellipse; (*see* TREDGOLD on Cast Iron, art. 155); in which case, the same formulæ are to be applied as when the beam is uniform throughout the length.

is the same as if half that load were applied at the middle point; (*see* TREDGOLD on Cast Iron, art. 102); therefore, by comparing it with the strain in the first case, it is

$$lw = 1700\, b d^2.$$

4. *When the beam is fixed at one end, and loaded at the other.* The effect of the straining force in this case, is obviously four times as great as in the first; therefore, by comparison, we have

$$lw = 212\tfrac{1}{2}\, b d^2.$$

5. *When the beam is fixed at one end, and loaded uniformly over the length.* Here the effect of the load is the same as if one half of it were collected at the extremity of the beam; hence, by comparison, we have

$$lw = 425\, b d^2.$$

6. *When the beam is supported at the ends, and the load increases as the distance from one of the supports.* The effect of the load in this case, is very nearly the same as we have stated it to be in the third; consequently, the same formula may be used in practice without sensible error: but mechanical writers have proved, that the real effect is equivalent to what would be produced if $\frac{16}{31}$ of the load were applied at the point of greatest strain; (*see* TREDGOLD on Cast Iron, art. 103); hence, by comparison, we get

$$lw = 1646\tfrac{7}{8}\, b d^2.$$

NOTE. When the breadth b is to the depth d as unity is to x, x representing any number at pleasure, the formulæ for the several cases become

$$\begin{aligned}
&1. \quad l w x = 850 d^3, \\
&2. \quad m n w x = 212\tfrac{1}{2} l d^3, \\
&3. \quad l w x = 1700 d^3, \\
&4. \quad l w x = 212\tfrac{1}{2} d^3, \\
&5. \quad l w x = 425 d^3, \\
&6. \quad l w x = 1646\tfrac{7}{8} d^3.
\end{aligned}$$

And when the beam is a square, or the breadth b equal to the depth d, $b d^2$ becomes d^3; therefore, if d^3 be substituted for $b d^2$ in each of the foregoing cases, the resulting equations apply to square beams when the direction of the straining force is parallel to the side: or, putting s for the side of the square, the formulæ for the several cases are, as below:

$$\begin{aligned}
&1. \quad l w = 850 s^3, \\
&2. \quad m n w = 212\tfrac{1}{2} l s^3, \\
&3. \quad l w = 1700 s^3, \\
&4. \quad l w = 212\tfrac{1}{2} s^3, \\
&5. \quad l w = 425 s^3, \\
&6. \quad l w = 1646\tfrac{7}{8} s^3.
\end{aligned}$$

From which it appears, that the transverse strength of a square beam, when the strain is parallel to the side, is directly as the cube of the side, and inversely as the length of the beam; but when the force acts

in the direction of the vertical diagonal, the equations must be derived from the following problem:

PROB. II. *To determine an expression for the side of a square beam of cast iron, to bear a given load, when the direction of the straining force coincides with the vertical diagonal; the length of the beam being given, and the elastic force of the material remaining perfect.*

We have seen by the note to the foregoing problem, that the transverse strength of a square beam, when the strain is parallel to the side, is directly as the cube of the side and inversely as the length of the beam; and the writers on mechanics have proved, that the strength of a beam in this position, is to the strength when the strain coincides with the diagonal, as 1 is to 1 divided by the square root of 2; (*see* TREDGOLD on Cast Iron, art. 80); for if s represent the side of a square, $s\sqrt{2}$ is the diagonal, and by comparing this with the first case in the note to the last problem, we have $1 : \frac{1}{\sqrt{2}} :: 850\,s^3 : 601\,s^3$, or, by taking the even number 600, which is sufficiently accurate for practical purposes, we obtain

1. *When the beam is supported at the ends, and loaded in the middle*,

$$l w = 600\,s^3.$$

2. *When the beam is supported at the ends, but the*

load not in the middle, we have, as in the second case of problem first,

$$m n w = 150 l s^3.$$

3. *When the beam is supported at the ends, and loaded uniformly over the length,*

$$l w = 1200 s^3.$$

4. *When the beam is fixed at one end, and loaded at the other,*

$$l w = 150 s^3.$$

5. *When the beam is fixed at one end, and loaded uniformly over the length,*

$$l w = 300 s^3.$$

6. *When the beam is supported at the ends, and the load increases as the distance from one of the supports,*

$$l w = 1162\tfrac{1}{2} s^3.$$

Prob. III. *To determine an expression for the diameter of a cylindric beam of cast iron, to bear a given load; the length of the beam being given, and the elastic force of the material remaining perfect.*

It is demonstrated by writers on mechanics, that the transverse strength of a cylindric beam is directly as the cube of the diameter and inversely as the length, and, moreover, that its strength is to the strength of its circumscribing square as 4·71 to 8; (*see* Tredgold on Cast Iron, art. 81); hence, by comparison, we have 8 : 4·71 : : 850 : 500 very

nearly; therefore, if d be put for the diameter of the cylinder, we obtain

1. *When the beam is supported at the ends, and loaded in the middle,*

$$l w = 500 \, d^3.$$

2. *When the beam is supported at the ends, but the load not in the middle,*

$$m n w = 125 \, l d^3.$$

3. *When the beam is supported at the ends, and loaded uniformly over the length,*

$$l w = 1000 \, d^3.$$

4. *When the beam is fixed at one end, and loaded at the other,*

$$l w = 125 \, d^3.$$

5. *When the beam is fixed at one end, and loaded uniformly over the length,*

$$l w = 250 \, d^3.$$

6. *When the beam is supported at the ends, and the load increases as the distance from one of the supports,*

$$l w = 968\tfrac{3}{4} \, d^3.$$

Prob. IV. *To determine an expression for the exterior diameter of a cylindrical tube of cast iron, to bear a given load; the thickness of metal and length of the tube being given, and the elastic force of the material remaining perfect.*

Let d be the exterior diameter of the tube, and t

the thickness of metal, then will $d-2t$ be the diameter of the hollow part: put p a fraction such, that $pd=d-2t$, then, by the first case of the last problem, the strength of a solid cylindric beam, having the diameter d, is as $500\,d^3$, and the strength of a beam having the diameter $p\,d$ is as $500p^3d^3$; consequently, the strength of the tube or fistular column, is as the difference of these expressions, that is, as $500d^3(1-p^3)$; but by reason of the increased resistance of the inner fibres, when the metal is expanded into a tube, the strength is accurately as $500d^3(1-p^4)$; hence, the general expression for the beam, when supported at the ends and loaded in the middle, is

$$lw=500d^3\,(1-p^4). \qquad \text{(A)}$$

This equation, when modified for the other cases of the problem, is applicable to the calculation of beams that have already been constructed; but in practice, when beams are required to be estimated by calculation, previously to their formation, it is preferable to assume some proportion between the parts, as between the exterior and interior diameters in the present instance. But the best proportion that can be assigned, is evidently that which gives the greatest strength with the least quantity of material, and the writers on the resistance of solids have shewn, that when a certain portion of metal is expanded into a tubular form, its strength is increased in the ratio of the diameters of its cross section, and

the greatest excess of strength that can safely be attained on account of the flaccidity of the material, is double of that which obtains in the solid state; (*see* TREDGOLD on Cast Iron, art. 83). Now, it is easy to shew, that when the strength of the tube is double that of the solid cylinder containing the same quantity of metal, the exterior diameter is to the interior nearly as 1 is to 0·7166, that is, $p = 0{\cdot}7166$; therefore, by substituting this value of p in the general expression marked (A), we obtain

1. *When the beam is supported at the ends, and loaded in the middle,*

$$lw = 368d^3.$$

2. *When the beam is supported at the ends, but the load not in the middle,*

$$mnw = 92\,l\,d^3.$$

3. *When the beam is supported at the ends, and loaded uniformly over the length,*

$$lw = 736d^3.$$

4. *When the beam is fixed at one end, and loaded at the other,*

$$lw = 92d^3.$$

5. *When the beam is fixed at one end, and loaded uniformly over the length,*

$$lw = 184d^3.$$

6. *When the beam is supported at the ends, and the load increases as the distance from one of the supports,*

$$lw = 713d^3.$$

PROB. V. *To determine an expression for the dimensions of a grooved beam of cast iron, to bear a given load; the length of the beam being given, and the elastic force of the material remaining perfect.*

Let b be the greatest breadth of the beam, or the breadth of the upper and lower parts, d the whole depth, B the breadth of the middle part, and D its depth, then, $b - \mathrm{B}$ and $d - \mathrm{D}$ are respectively the breadth and depth of the projections: take p and q two fractions such, that $p\,d = \mathrm{D}$ and $q\,b = b - \mathrm{B}$; then $p = \frac{\mathrm{D}}{d}$ and $q = \frac{b - \mathrm{B}}{b}$. Now, the strength of a beam of the present form, is evidently equal to the difference between the strength of a beam whose breadth is b and depth d, and that whose breadth is $q\,b$ and depth $p\,d$; that is, the strength of a grooved beam is as $850\,b\,d^2(1 - q\,p^2)$, which expression, when corrected for the effect of extension, becomes $850\,b\,d^2 (1 - q\,p^3)$. Hence, the general expression for a grooved beam, when supported at the ends, and loaded in the middle of its length, is

$$l\,w = 850\,b\,d^2(1 - q\,p^3). \qquad \text{(B)}$$

But in order that this equation may be rendered available in practice, it becomes necessary to assign particular values to the letters p and q, so that the proportions deduced from calculation, may give the greatest increase of strength with the least increase in the quantity of material. If p be taken equal 0·7 and q equal 0·625, fractions which answer very well

in practice; (*see* TREDGOLD on Cast Iron, art. 150); then, by substituting these values for p and q in the general expression marked (B), we obtain

1. *When the beam is supported at the ends, and loaded in the middle,*

$$l w = 668 \, b d^2.$$

2. *When the beam is supported at the ends, but the load not in the middle,*

$$m n w = 167 \, l b d^2.$$

3. *When the beam is supported at the ends, and loaded uniformly over the length,*

$$l w = 1336 \, b d^2.$$

4. *When the beam is fixed at one end, and loaded at the other,*

$$l w = 167 \, b d^2.$$

5. *When the beam is fixed at one end, and loaded uniformly over the length,*

$$l w = 334 \, b d^2.$$

6. *When the beam is supported at the ends, and the load increases as the distance from one of the supports,*

$$l w = 1294\tfrac{1}{4} \, b d^2.$$

PROB. VI. *To determine an expression for the dimensions of an open beam of cast iron, to bear a given load; the length of the beam being given, and the elastic force of the material remaining perfect.*

Let b be the breadth, d the whole depth, and D the depth of the open part, and put p a fraction such, that $p d = \mathrm{D}$; then will $p = \mathrm{D} \div d$. Now, the

strength of a beam of this form, is evidently equal to the difference between the strength of a beam, having the breadth b and depth d, and another having the breadth b and depth $p\,d$; that is, the strength of an open beam is as $850\,b\,d^2\,(1 - p^2)$, which, being modified for the effect of extension, becomes $850\,b\,d^2\,(1 - p^3)$. Hence, the general expression for an open beam, when supported at the ends and loaded in the middle, is

$$l\,w = 850\,b\,d^2\,(1 - p^3). \qquad \text{(C)}$$

Here, again, as in the two preceding problems, we have to assume a particular value of p. Let this assumed value be 0·7, as in Problem V. which being substituted in the expression marked (C), we obtain

1. *When the beam is supported at the ends, and loaded in the middle,*

$$l\,w = 558\,b\,d^2.$$

2. *When the beam is supported at the ends, but the load not in the middle,*

$$m\,n\,w = 139\tfrac{1}{2}\,l\,b\,d^2.$$

3. *When the beam is supported at the ends, and loaded uniformly over the length,*

$$l\,w = 1116\,b\,d^2.$$

4. *When the beam is fixed at one end, and loaded at the other,*

$$l\,w = 139\tfrac{1}{2}\,b\,d^2.$$

5. *When the beam is fixed at one end, and loaded uniformly over the length,*

$$l\,w = 279\,b\,d^2.$$

6. *When the beam is supported at the ends, and the load increases as the distance from one of the supports.*

$$lw = 1081\tfrac{1}{8}\,b\,d^2.$$

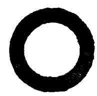
NOTE. The observations which we made in the note to Case 2 of Problem I. apply equally to the form of beams in this and Problem V. preceding.

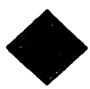
If the constants which we have derived for the several cases of each problem be compared with one another, it will be seen that they bear precisely the same relation among themselves as we stated in the premises, viz. that they are as the numbers 1, $\frac{1}{4}$, 2, $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{31}{16}$. Now, the constant for the first case of each problem, being obtained by considering the mechanical effect of the load when applied at the middle of the beam, or at the point of greatest strain, the others are derived from it, by simply multiplying by the preceding numbers; but the reader will more clearly perceive the relation, by tracing in detail the several cases of the first problem.

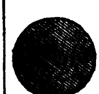
From the preceding formulæ, the particular circumstances respecting beams exposed to transverse strains from pressure or weight can be calculated, but for the convenience of reference and comparison, we shall exhibit the results in a tabular form, together with the section of the beam to which each class of equations more immediately applies.

NOTE. In the tabulated forms, we shall take the constants to the nearest unit only, omitting the fractions as being inconsiderable in practical cases.

Table of Formulæ for calculating the Strength and Dimensions of Cast Iron Beams, when exposed to Transverse Strains from Pressure or Weight.

	Rectangular Beams.	Form of Section.		Tubular Beams.	Form of Section.
1	$lw = 850\,bd^2$		1	$lw = 368\,d^3$	
2	$mnw = 212\,lbd^2$		2	$mnw = 92\,ld^3$	
3	$lw = 1700\,bd^2$		3	$lw = 736\,d^3$	
4	$lw = 212\,bd^2$		4	$lw = 92\,d^3$	
5	$lw = 425\,bd^2$		5	$lw = 184\,d^3$	
6	$lw = 1647\,bd^2$		6	$lw = 713\,d^3$	

	Square Beams.*	Form of Section.		Grooved Beams.	Form of Section.
1	$lw = 600\,s^3$		1	$lw = 668\,bd^2$	
2	$mnw = 150\,ls^3$		2	$mnw = 167\,lbd^2$	
3	$lw = 1200\,s^3$		3	$lw = 1336\,bd^2$	
4	$lw = 150\,s^3$		4	$lw = 167\,bd^2$	
5	$lw = 300\,s^3$		5	$lw = 334\,bd^2$	
6	$lw = 1162\,s^3$		6	$lw = 1294\,bd^2$	

	Cylindric Beams.	Form of Section.		Open Beams.	Form of Section.
1	$lw = 500\,d^3$		1	$lw = 558\,bd^2$	
2	$mnw = 125\,ld^3$		2	$mnw = 139\,lbd^2$	
3	$lw = 1000\,d^3$		3	$lw = 1116\,bd^2$	
4	$lw = 125\,d^3$		4	$lw = 139\,bd^2$	
5	$lw = 250\,d^3$		5	$lw = 279\,bd^2$	
6	$lw = 969\,d^3$		6	$lw = 1081\,bd^2$	

* This class of equations apply to square beams when the direction of the straining force coincides with the vertical diagonal.

To the reader who is acquainted with the reduction of algebraic equations, the formulæ as tabulated above are quite sufficient, the arrangement being so simple, that the value of the unknown term can be obtained at a single glance; but to those who are not acquainted with the elements of that science, a further illustration of the subject becomes necessary, and for this purpose we select the following examples:—

PROP. I. *The form and dimensions of the beam being given, to calculate the load that can be sustained, without destroying the elastic force of the material.*

Example 1. What weight will a rectangular beam of cast iron 2 inches broad, and 12 inches deep, sustain at the middle of its length, the distance between the supports being 20 feet?

Here, the beam is rectangular, supported at the ends and loaded in the middle; consequently, the formula of calculation is No. 1 Problem I. or No. 1 of the tabulated class for rectangular beams, viz.

$$lw = 850bd^2.$$

Now $b = 2$ inches, $d = 12$ inches, and $l = 20$ feet: let these numbers be substituted in the equation for b, d and l, and it becomes

$$20w = 850 \times 2 \times 144,$$

$$\text{or, } 20w = 244800;$$

$$\text{that is, } \quad w = \frac{244800}{20} = 12240 \text{ lbs.}$$

But the weight of the beam itself is 1600 lbs. and this is equivalent to 800 lbs. acting at the middle point; therefore, $12240 - 800 = 11440$ lbs. the load which the beam will sustain with safety.

Example 2. A cast iron beam, 6 inches square and 20 feet long, is supported at the ends with one of its diagonals vertical; what load can be sustained by it, when applied at a point 8 feet from one support and 12 feet from the other?

In this example the beam is square, supported at the ends, but the load not in the middle, and the direction of the straining force coincides with the vertical diagonal; this corresponds with the second case of the second problem, or No. 2 of the tabular class for square beams, viz.

$$mnw = 150\, l s^3.$$

Now $s = 6$ inches, $l = 20$ feet, $m = 8$ feet and $n = 12$ feet; let these values of s, l, m and n be substituted in the equation, and it becomes

$$8 \times 12 \times w = 150 \times 20 \times 6^3,$$

$$\text{or, } 96\, w = 648000;$$

$$\text{that is, } w = \frac{648000}{96} = 6750 \text{ lbs.}$$

But the weight of the beam is 2400 lbs. and this is equivalent to 1152 lbs. acting at the point where the load is applied; therefore, $6750 - 1152 = 5598$ lbs. the load which the beam will sustain, without having its elastic force destroyed.

Example 3. A solid cylindric beam of cast iron 6 inches diameter, is supported at both ends, and loaded uniformly over the length; what weight will it bear, the distance between the supports being 20 feet?

Here, the beam is cylindrical, supported at the ends and loaded uniformly over the length; this agrees with Case 3, Problem III. or No. 3 of the tabular class for cylindric beams: the formula is

$$l\,w = 1000 d^3.$$

Now $d = 6$ inches and $l = 20$ feet; let these numbers be substituted for d and l in the equation, and it becomes

$$20\,w = 1000 \times 6^3,$$

$$\text{or, } 20\,w = 216000;$$

$$\text{that is, } w = \frac{216000}{20} = 10800 \text{ lbs.}$$

The weight of the beam is 1885 lbs; therefore, 10800 − [redacted] = [redacted] lbs. the load that the beam will bear with safety.

Example 4. A tubular beam of cast iron, having its exterior diameter 6 inches, is fixed in a wall at one end, and loaded at the other; what weight will it bear, supposing its length to be 8 feet?

In this example the beam is tubular, fixed at one end and loaded at the other; this agrees with Case 4, Problem IV. or No. 4 of the tabulated class for tubular beams: the formula is

$$lw = 92 d^3.$$

Now $d = 6$ inches and $l = 8$ feet; let these numbers be substituted for d and l in the equation, and it becomes

$$8\,w = 92 \times 6^3,$$

$$\text{or, } 8\,w = 19872;$$

that is, $w = \frac{19872}{8} = 2484$ lbs. including the weight of the tube; or rather, a weight which being applied at the extremity will produce the same effect.

NOTE. The preceding result, arising from the tabular equation, is obtained on the supposition that p is a constant quantity, and equal to the fraction 0·7166, in which case, the diameters have constantly the same relation to each other; but it may frequently happen, that the ratio of the diameters varies from that for which the tabulated form has been derived, and then recourse must be had to the general equation (A), Problem IV. where p is always the quotient that obtains, when the interior diameter is divided by the exterior diameter of the tube.

We may also remark, that on account of the plan of our arrangement, we have been compelled, in the present instance, to illustrate a case that is not likely often to occur in practice, because tubes are seldom used in the manner implied in the example; but our present object being to shew how the equations are to be applied, we see no

reason for deviating from the plan that we have laid down; the skilful mechanic, however, will easily discriminate the useful practical applications.

Example 5. A grooved beam of cast iron, ~~4~~ 6/ inches broad and 18 inches deep, is fixed in a wall at one end, and loaded uniformly over the length; what weight will it bear, supposing its length to be 8 feet?

Here, the beam is grooved, fixed at one end, and loaded uniformly over the length; this answers to Case 5, Problem V. or No. 5 of the tabular class for grooved beams: the formula is

$$lw = 334bd^2.$$

But $b = 6$ inches, $d = 18$ inches, and $l = 8$ feet; let these numbers be substituted for b, d and l in the equation, and it becomes

$$8w = 334 \times 6 \times 18^2,$$

$$\text{or, } 8w = 649296;$$

$$\text{that is, } w = \frac{649296}{8} = 81162 \text{ lbs.}$$ the whole load that the beam ought to sustain; but the weight of the beam is 1080 lbs. therefore, the weight with which it ought to be loaded is 81162 − ~~1080~~ = 80082 lbs. 35/ 62/ 010/

NOTE. The foregoing result arises from the supposition that p and q are constant quantities, and equal respectively to the fractions 0·7 and 0·625, in which case, the parts of the beam have always the

same relation to each other, whatever may be the magnitude of those parts; it does, however, frequently happen that no such relation exists between the parts, and then the solution must be sought from the general equation (B) Problem V. where p is the depth of the grooved or middle part, divided by the depth of the beam, and q the difference between unity and the breadth of the groove, divided by the breadth of the beam.

Example 6. An open beam of cast iron, 3 inches broad, 18 inches deep, and 20 feet long, is supported at the ends, and loaded in such a manner that the weight increases as the distance from one of the supports.

In this example, the beam is open, supported at the ends, and the load increasing as the distance from one of the supports; this corresponds to Case 6, Problem VI. or No. 6 of the tabular class for open beams: the formula is

$$lw = 1081\,bd^2.$$

Now $b = 3$ inches, $d = 18$ inches, and $l = 20$ feet; let these values be substituted for b, d and l in the equation, and it becomes

$$20\,w = 1081 \times 3 \times 18^2,$$

$$\text{or, } 20\,w = 1050732;$$

that is, $w = \dfrac{1050732}{20} = 52536\tfrac{3}{5}$ lbs. = the whole load sustained, including the weight of the beam.

It is presumed, that the preceding examples will be found sufficient for exemplifying the method of computing the load, when the dimensions of the beam are known: we shall next shew, in what manner the dimensions of breadth and depth are to be calculated, the length and the load being given, together with the breadth when the depth is wanted, or the depth when the breadth is wanted; it being necessary, that three things should be given when four are concerned in the inquiry, or two given when only three are concerned.

In the application of beams to practice, the length is always limited by the circumstances of situation, and the load to be sustained is estimated from the conditions of construction; therefore, in what follows, we shall consider these data as being known.

PROP. II. *The form and length of the beam being given, to calculate the breadth and depth to sustain a given load, without destroying the elastic force of the material.*

Example 1. What must be the breadth of a cast-iron rectangular beam, to bear a weight of 12240 lbs. in the middle of its length, when supported at the ends, the distance between the supports being 20 feet, and depth of the beam 12 inches?

The beam is rectangular, supported at the ends

and loaded in the middle; this agrees with Case 1, Problem I. or No. 1 of the class for rectangular beams: the formula is

$$lw = 850\,bd^2.$$

But $w = 12240$ lbs. $l = 20$ feet, and $d = 12$ inches; let these numbers be substituted in the equation for w, l, d, and it becomes

$$12240 \times 20 = 850 \times 12^2 \times b,$$

that is, $244800 = 122400\,b$;

therefore, $b = \frac{244800}{122400} = 2$ inches, the breadth sought.

Let us next suppose the breadth is given equal 2 inches, and that it is required to calculate the depth; substitute the numbers for the weight and length as before, only, instead of the depth, we have here to substitute the breadth, and it becomes

$$244800 = 1700d^2,$$

$$\text{or, } d^2 = \frac{244800}{1700} = 144;$$

hence $d = \sqrt{144} = 12$ inches, the depth sought.

Example 2. Find the side of a square beam, under the conditions specified in Case 2, Problem II. the length being 20 feet, and a load of 6750 lbs. being applied at a point 8 feet from one support and 12 feet from the other.

The formula is No. 2 of the class for square beams, viz.

$$mnw = 150ls^3.$$

Here $w = 6750$ lbs. $l = 20$ feet, $m = 8$ feet, and $n = 12$ feet; let these values be substituted for w, l, m and n, and our equation becomes

$$8 \times 12 \times 6750 = 150 \times 20 \times s^3;$$

That is, $648000 = 3000\ s^3$,

$$\text{or, } s^3 = \frac{648000}{3000} = 216;$$

therefore $s = \sqrt[3]{216} = 6$ inches, the side of the square sought, and $6\sqrt{2} = 8{\cdot}4852$ inches, is the diagonal, in the direction of which the force acts.

Example 3. Find the diameter of a cylindric beam, under the conditions specified in Case 3, Problem III. its length being 20 feet, and a load of 10800 lbs. uniformly distributed over the length. The formula is No. 3 of the class for cylindric beams, viz.

$$lw = 1000d^3.$$

In this example, $w = 10800$ lbs. and $l = 20$ feet; let these values be substituted for w and l in the equation, and it becomes

$$10800 \times 20 = 1000\ d^3,$$

$$\text{that is, } d^3 = \frac{216000}{1000} = 216,$$

or $d = \sqrt[3]{216} = 6$ inches, the diameter sought.

Example 4. Find the exterior diameter of a tubular beam, under the conditions specified in

Case 4, Problem IV. its length being 8 feet, and having a load of 2484 lbs. applied at its extremity. The formula is No. 4 of the class for tubular beams, viz.

$$l\,w = 92\,d^3.$$

Here $l = 8$ feet and $w = 2484$ lbs. let these numbers be substituted for l and w in the equation, and it becomes

$$8 \times 2484 = 92\,d^3;$$

$$\text{that is, } 19872 = 92\,d^3,$$

$$\text{or } d^3 = \frac{19872}{92} = 216;$$

hence $d = \sqrt[3]{216} = 6$ inches, the diameter sought.

Note. The observations in the note to Example 4, Prop. I. will apply in the same manner to this example.

Example 5. Find the breadth of a grooved beam of cast iron, to bear a load of 81162 lbs. equally diffused over its length, under the conditions specified in Case 5, Problem V. the length of the beam being 8 feet, and its depth 18 inches.

The formula is No. 5 of the class for grooved beams, viz.

$$l\,w = 334\,b\,d^2.$$

Here $w = 81162$ lbs. $l = 8$ feet, and $d = 18$ inches. Substitute these numbers in the equation for w, l and d, and it becomes

$$81162 \times 8 = 334 \times 18^2 \times b;$$

that is, $649296 = 108216\ b$,

$$\text{or } b = \frac{649296}{108216} = 6 \text{ inches, the breadth}$$

required.

Again, let us suppose that the breadth is given equal to 6 inches, and that it is required to calculate the depth. Substitute the numbers for the load and length, as before; only, instead of the depth we have to substitute the breadth, and our equation becomes

$$649296 = 2004\, d^2,$$

$$\text{or } d^2 = \frac{649296}{2004} = 324;$$

hence $d = \sqrt{324} = 18$ inches, the depth sought. (*See* Note to Example 5, Prop. I.)

Example 6. Find the breadth of an open beam, under the conditions described in Case 6, Problem VI. its depth being 18 inches, length 20 feet, and load $52536\frac{3}{5}$ lbs.

The formula is No. 6 of the class for open beams, viz.

$$lw = 1081\, b d^2.$$

In this example, $l = 20$ feet, $d = 18$ inches, and $w = 52536\frac{3}{5}$ lbs. let these values of l, d and w be substituted in the equation, and it becomes

$$52536\tfrac{4}{5} \times 20 = 1081 \times 18^2 \times b;$$

that is, $1050732 = 350244\ b$,

or $b = \frac{1050732}{350244} = 3$ inches, the breadth required.

Let us now suppose that the breadth is given equal to 3 inches, and it is required to find the depth. Substitute for the length and the load as before, but instead of the depth substitute the breadth, and the equation is

$$1050732 = 3243\ d^2;$$

$$\text{that is, } d^2 = \frac{1050732}{3243} = 324.$$

Hence $d = \sqrt{324} = 18$ inches, the depth sought.

It is presumed that what we have now done, will be found sufficient to enable the attentive reader to perceive the manner in which the formulæ generally are to be applied; but, to render the work more extensively useful, we shall give a table of constants, to be substituted in the several classes of the preceding equations, for calculating the strength and dimensions of wrought iron and wooden beams, such as are usually employed for architectural and mechanical purposes.

Table of Constants for calculating the Strength and Dimensions of Wrought Iron and Wooden Beams, when exposed to a Transverse Strain.

	Ash.	Beech.	Elm.	Fir, red or yellow.	Fir, white.	Oak.	Pine.	Wrought Iron.
				Rectangular Beams.				
1	196	128	178	255	196	212	212	952
2	49	32	45	64	49	53	53	238
3	392	256	356	510	392	425	425	1904
4	49	32	45	64	49	53	53	238
5	98	64	90	128	98	106	106	476
6	379	247	346	494	379	412	412	1845
				*Square Beams.**				
1	138	90	126	180	138	150	150	672
2	35	23	32	45	35	38	38	168
3	276	180	252	360	276	300	300	1344
4	35	23	32	45	35	38	38	168
5	70	46	64	90	70	75	75	336
6	267	174	244	349	267	291	291	1291
				Cylindrical Beams.				
1	115	75	105	150	115	125	125	560
2	29	19	26	38	29	31	31	140
3	230	150	210	300	230	250	250	1120
4	29	19	26	38	29	31	31	140
5	58	38	52	76	58	62	62	280
6	223	145	203	291	223	242	242	1085

	Ash.	Beech.	Elm.	Fir, red or yellow.	Fir, white.	Oak.	Pine.	Wrought Iron.
				Tubular Beams.				
1	85	55	77	110	85	92	92	412
2	21	14	19	28	21	23	23	103
3	169	110	154	220	169	184	184	824
4	21	14	19	28	21	23	23	103
5	42	28	38	56	42	46	46	206
6	164	107	150	214	164	178	178	998
				Grooved Beams.				
1	154	100	140	200	154	167	167	748
2	38	25	35	50	38	42	42	187
3	307	200	280	400	307	334	334	1496
4	38	25	35	50	38	42	42	187
5	76	50	70	100	76	84	84	374
6	298	194	272	388	298	323	323	1449
				Open Beams.				
1	128	84	117	168	128	139	139	625
2	32	21	29	42	32	35	35	156
3	257	168	234	336	257	279	279	1249
4	32	21	29	42	32	35	35	156
5	64	42	58	84	64	70	70	312
6	249	162	227	324	249	270	270	1211

* This class of constants apply to square beams when the straining force acts in the direction of the vertical diagonal, but when it acts parallel to the side, the constants for rectangular beams must be employed.

The numbers in this Table have been calculated from the relative strength of the several materials as compared to cast iron, and the following example will shew the manner in which they are to be used.

A rectangular beam of American pine, 3 inches broad, 18 feet long, and supported at the ends, is required to bear 13440 lbs. uniformly distributed over the length: what must be the depth?

The formula for a cast-iron beam so circumstanced, is No. 3 of the class for rectangular beams, viz.

$$lw = 1700\, b d^2,$$

and the constant for the same number and class of pine-beams in the preceding Table, is 425. Let this be used instead of 1700, and substitute the given weight, length and breadth, and we have

$$241920 = 1275\, d^2,$$

$$\text{or } d = \sqrt{\frac{241920}{1275}} = 13\tfrac{3}{4} \text{ inches nearly,}$$

the depth of the beam.

In the foregoing pages we have shewn the method of calculating the particulars of the most common forms of cast iron beams, when exposed to transverse strains from pressure or weight; and have, by means of a table of constants, applied the same principles to calculate the particulars of similar beams of wrought iron, and several species of

wood: yet there are other forms, which, although less useful, may occasionally occur in practice, such as the following; viz., those having the transverse sections *elliptical* and *triangular*, and that form so commonly used, having its transverse section in form of the letter T. We have deferred treating on these forms till now, not merely because they did not range with the general plan, but more especially as the elliptical and triangular sections, unless compelled by the circumstances of situation, are seldom resorted to for bearing purposes, and the third or T form being objectionable as respects economy. But that nothing may be wanting to satisfy the general reader, we shall illustrate the above forms in the following problems.

PROBLEM VII. *To determine an expression for the dimensions of an elliptical beam of cast iron, to bear a given load; the length of the beam being given, and the elastic force of the material remaining perfect.*

It is easy to see that this problem admits of two cases:—

1st, When the strain is in the direction of the conjugate axis.

2d, When the strain is in the direction of the transverse axis.

First, then, when the strain is in the direction

of the conjugate axis. By comparing equations 16 and 17, (arts. 81 and 82, Tredgold on Cast Iron), it appears, that if the diameter of a cylindric beam be equal to the conjugate axis of the cross section of an elliptic beam, the strength of the latter is to that of the former, as the transverse axis of the elliptic section is to the conjugate axis: hence, for the several cases of the problem, as respects the position of the load and the manner of supporting or fixing the beam, we have the following equations:—

1. $lw = 500tc^2$,
2. $mnw = 125ltc^2$,
3. $lw = 1000tc^2$,
4. $lw = 125tc^2$,
5. $lw = 250tc^2$,
6. $lw = 968\frac{3}{4}tc^2$.

Section.

Where t is the transverse, and c the conjugate axis of the elliptic section.

Second, when the strain is in the direction of the longer axis of the ellipse, the formula for the several cases will be as under:—

1. $lw = 500ct^2$,
2. $mnw = 125lct^2$,
3. $lw = 1000ct^2$,
4. $lw = 125ct^2$,
5. $lw = 250ct^2$,
6. $lw = 968\frac{3}{4}ct^2$.

Section.

Where t and c are the transverse and conjugate axes, as before.

PROBLEM VIII. *To determine an expression for the dimensions of a triangular beam of cast iron, to bear a given load; the length of the beam being given, and the elastic force of the material remaining perfect.*

Mr. TREDGOLD, in his *Essay on Cast Iron*, (art. 85[a]), has shewn, that the strength of a triangular beam is to that of its circumscribing rectangular one, as 339 to 1000; but the constant for the first case of the rectangular beam is 850, (see Case 1, Problem I.): therefore it is

$$1000 : 850 :: 339 : 288.$$

Hence the formula for the several cases of the triangular beam are,

1. $l w = 288 b d^2$,
2. $m n w = 72 l b d^2$,
3. $l w = 576 b d^2$,
4. $l w = 72 b d^2$,
5. $l w = 144 b d^2$,
6. $l w = 558 b d^2$.

Section.

Where b is the base, and d the perpendicular depth of the triangular section.

Section

(For 288. put 170.

Problem IX. *To determine an expression for the dimensions of a cast iron beam, to bear a given load; the length of the beam being given, its transverse section in form of the letter T, and the elastic force of the material remaining perfect.*

Let p and q be assumed as in Problem V.; then, from principles nearly similar to those employed for calculating the general equation to that problem, we obtain

$$lw = 850\,bd^2\left\{\frac{1-q}{\left(1+\frac{\sqrt{1-q}}{\sqrt{1-p^3 q}}\right)^2}\right\}$$

This expression is very prolix, but it will not admit of a simpler arrangement; and, in fact, it is not difficult to reduce, for the numerator of the general fraction being the same as the numerator of the fraction under the radical sign, renders the reduction somewhat more easily effected than it would be if they were different quantities.

By assuming particular values for p and q, the formulæ for the several cases of the problem will be greatly simplified. Mr. Tredgold makes $p = \cdot 5$ and $q = \cdot 75$; (*see* his *Essay on Cast Iron*, art. 85[b]); in which case the strength of a beam of this form is to that of its circumscribing rectangle, in the ratio of 5 to 12: therefore

it is 12 : 5 : : 850 : 354. From which we get

1. $l\,w = 354\,b\,d^2$,
2. $m\,n\,w = 88\tfrac{1}{2}\,l\,b\,d^2$,
3. $l\,w = 708\,b\,d^2$,
4. $l\,w = 88\tfrac{1}{2}\,b\,d^2$,
5. $l\,w = 177\,b\,d^2$,
6. $l\,w = 686\,b\,d^2$.

Section.

Where b is the whole breadth, and d the whole depth of the section.

We abstain from giving examples to these problems, because, what we have done in the former part of the work, is quite sufficient to enable the intelligent reader to reduce any of the cases which we have just illustrated.

ON

THE DEFLEXION AND STIFFNESS

OF

CAST IRON BEAMS

EXPOSED TO TRANSVERSE STRAINS.

PROBLEM X. *To determine an expression for the deflexion of a uniform rectangular beam of cast iron, when the strain is equal to the elastic force of the material, or, according to* TREDGOLD, 15300 *lbs. on a square inch.*

It is sufficiently proved by experiment, (*see* TREDGOLD on Cast Iron, arts. 45 and 50), that when a beam is bent or deflected by means of a certain strain, the quantity of deflexion is directly proportional to the force that produces it, while the elastic force of the material remains perfect; hence, by an inference similar to that employed in the solution of the first problem, we find that the

square of the length in feet, divided by the depth in inches drawn into the deflexion, is a constant quantity; therefore, in a beam whose dimensions are known, if the deflexion produced by a given force be correctly ascertained, the deflexion for other beams similarly circumstanced can easily be calculated.

For, let D, L and Δ be the depth, length, and deflexion used in the experiment, and d, l and δ the depth, length, and deflexion for which the calculation is made; then, from the foregoing inference, we have.

$$\frac{L^2}{D\Delta}=\frac{l^2}{d\delta}$$

And this, by exterminating the fractions, becomes

$$d\delta L^2=D\Delta l^2.$$

Now, by Mr. Tredgold's experiment, (*see* TREDGOLD on Cast Iron, art. 45), it appears, that a load of 300 lbs. on the middle of a beam one inch square, and thirty-four inches between the supports, produced a deflexion of ·16 of an inch, while the elastic force of the material remained unimpaired. Let these dimensions be substituted for D, L and Δ in the preceding equation, and taking the length in feet, we obtain for the deflexion as follows:—

1. *When the beam is supported at the ends, and loaded in the middle.*

$$d\delta=\cdot02\,l^2.$$

2. *When the beam is supported at the ends, but the load not in the middle.*

It has been shewn, in the second case of the first problem, that the strain at the point where the load is applied, is proportional to the rectangle of the segments, into which the length of the beam is divided at that point; and we have stated above, that the deflexion is proportional to the force that produces it, while the strain is within the elastic power of the material; hence, by equality of ratios, the deflexion is proportional to the rectangle of the segments into which the length of the beam is divided at the point of strain. Let m and n represent these segments, or the respective distances of the straining force from the points of support; then, since the deflexion produced by the load in the middle of the beam, where the rectangle of the segments is $\frac{1}{4}l^2$, has been shewn by the last case to be as $\cdot 02\,l^2$, we have

$$\tfrac{1}{4}l^2 : mn :: \cdot 02\,l^2 : \cdot 08\,mn.$$

Hence, when the beam is supported at the ends, but the load not in the middle, the equation for the deflexion is

$$d\delta = \cdot 08\,mn.$$

3. *When the beam is supported at the ends, and loaded uniformly over the length.*

It is demonstrated by writers on the resistance

of solids, that a load uniformly distributed over the length of a beam, produces the same deflexion as if five-eighths of that load were collected at the middle point. (*See* TREDGOLD on Cast Iron, arts. 18 and 217). Now, in the third case of the first problem it is shewn, that the load which a rectangular beam will support, when uniformly distributed over the length, is $\frac{1700\,b\,d^2}{l}$; and by the first case of the same problem, the load supported in the middle is $\frac{850\,b\,d^2}{l}$; and we have shewn, in the first case preceding, that the deflexion produced by the load in the middle is $\frac{\cdot 02\,l^2}{d}$. But five-eighths of $\frac{1700\,b\,d^2}{l}$ is $\frac{8500\,b\,d^2}{8\,l}$; therefore, we have

$$\frac{850\,b\,d^2}{l} : \frac{8500\,b\,d^2}{8\,l} :: \frac{\cdot 02\,l^2}{d} : \frac{\cdot 025\,l^2}{d}.$$

Hence the equation for the deflexion of a beam, when loaded uniformly over the length, is

$$d\,\delta = \cdot 025\,l^2.$$

4. *When the beam is fixed at one end, and loaded at the other.* It has been stated, in the fourth case of the first problem, that the strain on a beam fixed at one end and loaded at the other, is quadruple the strain on a beam supported at the ends and loaded in the middle, and the deflexion is proportional to the strain; therefore, if ϕ be the angle of deflexion,

and R the quotient that arises from the division of the fixed part by the projecting part of the beam (the length of which is l), then the general expression for the deflexion is

$$d\delta = \cdot 08\, l^2 (1 + \mathrm{R} \cos.\varphi).$$

But, since φ, the angle of deflexion, is always very small in practical cases, its cosine will not differ sensibly from unity; in which case, the preceding expression becomes

$$d\delta = \cdot 08\, l^2 (1 + \mathrm{R}).$$

If the fixed part of the beam be greater than the projecting part, or such, that its flexure is very small, we have

$$d\delta = \cdot 08\, l^2.$$

If the fixed part be equal to the projecting part, or $\mathrm{R} = 1$, we have

$$d\delta = \cdot 16\, l^2.$$

It seems unnecessary to calculate the deflexion for the other two cases of the problem, viz.

When the beam is fixed at one end and loaded uniformly over the length; and when the beam is supported at the ends, and the load increases as the distance from one of the supports; and moreover, we may here remark, that the equations for the deflexion which we have just investigated, are equally applicable to the same cases of beams, of the several forms of section given in the table, at page 18, when

the depth throughout the length is uniform. In a tabulated form they are as follow:

Table of Formulæ for calculating the Deflexions of Cast Iron Beams, when exposed to Transverse or Cross Strains.

Table of Deflexions from Cross Strains.	
1	$d\,\delta = \cdot 02\,l^2,$
2	$d\,\delta = \cdot 08\,m\,n,$
3	$d\,\delta = \cdot 025\,l^2,$
4	$\left\{ \begin{array}{l} d\,\delta = \cdot 08\,l^2\,(1+\mathrm{R}), \\ d\,\delta = \cdot 08\,l^2, \\ d\,\delta = \cdot 16\,l^2. \end{array} \right\}$

Where the bracketed expressions are those for the several relations of the fixed and projecting parts of the beam, mentioned in Case 4.

The rule for calculating the deflexions of beams in the several cases, may be enunciated in general terms, as follows.

GENERAL RULE.

Multiply the square of the length, or the rectangle of the segments at the point of strain, in feet, by the constant number for the particular case, and divide

the product by the depth of the beam in inches, and the quotient will be the deflexion required.

One or two examples, it is presumed, will be found sufficient to exemplify the method of applying the formulæ.

Example 1. A uniform rectangular beam of cast iron, 20 inches deep and 25 feet between the supports, is strained in the middle to the extent of its elastic force: what is the deflexion?

The conditions of the beam, as specified in the question, correspond to the first case of the 10th problem, or No. 1 of the tabular formulæ for the deflexions from cross strains, the expression for which is

$$d\delta = \cdot 02 l^2.$$

Here $d = 20$ inches, and $l = 25$ feet; let these numbers be substituted for d and l in the equation above, and it becomes

$$20\delta = \cdot 02 \times 25^2,$$

$$\text{or, } \delta = \frac{\cdot 02 \times 625}{20} = \cdot 625 \text{ of an inch,}$$

the deflexion required.

Let us now suppose, that the dimensions of the beam remain the same, but that the load, instead of being in the middle, is placed at a point 9 feet from one support and 16 feet from the other; what then is the deflexion?

Here then $d = 20$ inches, and $l = 25$ feet, as

before; and, moreover, $m = 9$ and $n = 16$ feet; let these numbers be substituted for d, l, m and n in No. 2 of the preceding table, and it becomes

$$20\delta = \cdot 08 \times 9 \times 16,$$

$$\text{or, } \delta = \frac{11\cdot 52}{20} = \cdot 576 \text{ of an inch, for}$$

the deflexion required.

Example 2. Required the deflexion at the extremity of a uniform beam of cast iron, 12 inches deep and 9 feet long, 3 feet of which is fixed in a wall, and a load applied at the other extremity; the strain being equal to the elastic force of cast iron?

The conditions of the beam, as described in this example, correspond to the fourth case of the 10th problem, or to the first of the bracketed expressions, No. 4, of the tabular equations for the deflexions from cross strains; the equation is

$$d\delta = \cdot 08 l^2 (1 + R).$$

Now, $d = 12$ inches, $l = 6$ feet, and $R = \frac{3}{6} = \cdot 5$; therefore our equation becomes, by substitution,

$$12\delta = \cdot 08 \times 6^2 \times 1\cdot 5,$$

$$\text{or, } \delta = \frac{\cdot 08 \times 36 \times 1\cdot 5}{12} = \cdot 36 \text{ of an inch,}$$

the deflexion sought.

We may here observe, that the same degree of deflexion would occur in the *square*, the *cylindric*, the *tubular*, the *grooved*, and the *open* beams, if placed in the same situation, and under the same

circumstances, but the straining force would be different in them all; and a like remark applies to the other cases.

NOTE. The *rectangular*, the *grooved*, and the *open* beams sometimes, with a view to economy, have the outline of the depth modified, so as to form a parabola or an ellipsis; in which case, the formulæ for calculating the strength and dimensions are the very same, as when the beams are uniform in breadth and depth, but the expressions for the deflexion are somewhat different, as follows:

FOR THE PARABOLIC FORM.

When the beam is supported at the ends, and loaded in the middle,

$$d\delta = \cdot 04\, l^2.$$

When the beam is fixed at one end, and loaded at the other,

$$d\delta = \cdot 16\, l^2 (1 + R).$$

FOR THE ELLIPTIC FORM.

When the beam is supported at the ends, and loaded in the middle,

$$d\delta = \cdot 0257\, l^2.$$

When the beam is fixed at one end, and loaded at the other,

$$d\delta = \cdot 1028\, l^2 (1 + R).$$

There are several other forms of beams that

sometimes occur in practice, which it may be useful to consider in this place, such as the following, viz.

Beams having the breadth uniform, and the depth at the extremities one half the depth at the middle. And *Beams having the depth uniform and the breadth bounded by a triangle.*

The equations for calculating the deflexions of these forms are as follow:

When the beams are supported at the ends, and loaded in the middle; we have for the deflexions, respectively,

$$d\delta = \cdot 0327\, l^2,$$
$$d\delta = \cdot 03\, l^2.$$

When the beams are fixed at one end, and loaded at the other, the respective deflexions are

$$d\delta = \cdot 13\, l^2 (1 + \mathrm{R}).$$
$$d\delta = \cdot 12\, l^2.$$

The flexure of the fixed part of the beam in the last expression is supposed to be very small; for which reason, the parenthetical member of the expression has been omitted.

If the constant numbers in the several expressions which we have derived for calculating the deflexions of cast iron beams be multiplied respectively by the numbers that mark the relative extensibility to cast iron, of the several materials mentioned in the following table, the results will be

the constants to be applied in calculating the deflexions of beams constructed of such materials accordingly; the forms of the equations remaining otherwise unaltered.

The following is a table of the relative extensibility of the materials therein named, compared with that of cast iron as unity.

Table of the relative Extensibility of Materials.	
Ash	2·6
Beech	2·1
Elm	2·9
Fir, red or yellow	2·6
Fir, white..................	2·4
Oak, English	2·8
Pine, American	2·9
Wrought Iron	0·86

The following example will shew the method of applying the numbers in the preceding table.

Example 1. A beam of American pine, 16 inches deep and 22 feet long, is supported at the ends, and loaded uniformly over the length. What is the deflexion when strained to the extent of its elastic force?

The conditions of the beam, as specified in the

example, correspond to the third case of Problem X. for which the expression is

$$d\delta = \cdot 025\, l^2.$$

Now, the relative extensibility for American pine is 2·9, and the constant in the formula is ·025; therefore, $\cdot 025 \times 2\cdot 9 = \cdot 0725 =$ the constant for American pine; hence, the formula for the deflexion is

$$d\delta = \cdot 0725\, l^2.$$

But $d = 16$ inches, and $l = 22$ feet; let these numbers be substituted for d and l in the preceding equation, and it becomes

$$16\,\delta = \cdot 0725 \times 22^2,$$

$$\text{or, } \delta = \frac{\cdot 0725 \times 484}{16} = 2\cdot 2 \text{ inches nearly,}$$

the deflexion.

PROB. XI. *To determine an expression for the dimensions of a uniform rectangular beam of cast iron, to bear a given load, and resist a given deflexion; the length of the beam being given.*

We have shewn by the inference in the solution of the last problem, that *the square of the length in feet, divided by the depth in inches drawn into the deflexion,* is a constant quantity, from which property, by using the experimental numbers (*see* TREDGOLD on Cast Iron, art. 45), we found the expression for the deflexion of a beam supported at the ends and loaded in the middle to be $d\delta = \cdot 02\, l^2$,

(*see* Case 1, Problem X.) or more accurately $7225\,d\delta = 144\,l^2$; and, by the corresponding case of the first problem, the expression for the load capable of producing that deflexion is $lw = 850\,bd^2$. Now the deflexion is directly proportional to the strain that produces it (*see* Case 2, Prob. X.); hence, if we put D for any other deflexion, we have

$$\frac{144\,l^2}{7225\,d} : \mathrm{D} :: \frac{850\,bd^2}{l} : \frac{42648\,bd^3\mathrm{D}}{l^3} = \text{the load that}$$

will produce the deflexion D; therefore, the expression for the dimensions is,

1. *When the beam is supported at the ends, and loaded in the middle,*

$$l^3 w = 42648\,bd^3\mathrm{D}.$$

2. *When the beam is supported at the ends, but the load not in the middle.* In the second case of the tenth problem, the expression for the deflexion of a beam thus circumstanced, is shewn to be $d\delta = \cdot 08\,mn$, or more accurately, $7225\,d\delta = 576\,mn$; and by the second case of the first problem, the expression for the load to produce that deflexion is $mnw = 212\tfrac{1}{2}\,lbd^2$. Let D be any other deflexion, and we have

$$\frac{576\,mn}{7225\,d} : \mathrm{D} :: \frac{425\,lbd^2}{2\,mn} : \frac{2665\,lbd^3\,\mathrm{D}}{m^2 n^2} = \text{the load that}$$

will produce the deflexion D; hence, the expression for the dimensions is

$$m^2 n^2 w = 2665\,lbd^3\mathrm{D}.$$

3. *When the beam is supported at the ends, and loaded uniformly over the length.* We have shewn in the third case of the tenth problem, that the expression for the deflexion of a beam supported at the ends, and loaded uniformly over the length, is $d\delta = \cdot 025\, l^2$, or more correctly, $1445\, d\delta = 36\, l^2$; and by the third case of the first problem, the expression for the load to produce that deflexion is $l w = 1700\, b\, d^2$. Let D be any other deflexion, and we have

$\frac{36\, l^2}{1445\, d} : D :: \frac{1700\, b\, d^2}{l} : \frac{68236\, b\, d^3\, D}{l^3}$ = the load that will produce the deflexion D; therefore, the expression for the dimensions is

$$l^3 w = 68236\, b\, d^3 D.$$

4. *When the beam is fixed at one end, and loaded at the other.* It has been shewn in the fourth case of the tenth problem, that the general expression for the deflexion of a beam, when fixed at one end, and loaded at the other, is $d\delta = \cdot 08\, l^2(1 + R \cos. \phi)$, or more accurately, $7225\, d\delta = 576\, l^2(1 + R \cos. \phi)$; and by the fourth case of the first problem, the expression for the load to produce that deflexion is $l w = 212\frac{1}{2}\, b\, d^2$. Let D be any other deflexion, and we have

$\frac{576\, l^2\,(1 + R \cos. \phi)}{7225\, d} : D :: \frac{425\, b\, d^2}{2\, l} : \frac{2665\, b\, d^3\, D}{l^3\,(1 + R \cos. \phi)}$

= the load that will produce the deflexion D; hence the expression for the dimensions is

$l^3 w = \frac{2665\, b\, d^3\, D}{1 + R \cos.\, \varphi}$; or, because φ the angle of deflexion, is always very small in practical cases, its cosine will not differ sensibly from unity, in which case the expression becomes

$$l^3 w = \frac{2665\, b\, d^3\, D}{1 + R}.$$

Where R is the quotient that arises, when the fixed part of the beam is divided by the projecting part, as we have before remarked in Case 4 of Problem X.*

NOTE. When the beam is square, or the breadth b equal to the depth d, $b d^3$ becomes d^4; or if s represent the side of the square, $s^4 = d^4$; therefore, if s^4 be substituted for $b d^3$ in each of the preceding cases, the formulæ for *square* beams, when the direction of the straining force is parallel to the side, will be as follows:

1. $l^3 w = 42648\, s^4 D$,
2. $m^2 n^2 w = 2665\, l\, s^4 D$,
3. $l^3 w = 68236\, s^4 D$,
4. $l^3 w = \frac{2665\, s^4\, D}{1 + R}$.

We have stated in the detail of the tenth problem

* If the constants in these several cases be compared among themselves, it will be found that they are as the numbers 1, $\frac{1}{16}$, $\frac{8}{5}$ and $\frac{1}{16}$; and the same relation holds, whatever may be the form of the beam under consideration. This, therefore, renders the derivation of the formulæ a matter of great simplicity.

(*see* also TREDGOLD on Cast Iron, art. 173), that the expressions there given for calculating the deflexions for the several cases of a uniform rectangular beam, apply also to beams of the forms of section given in the table of formulæ, at p. 18; but since the constants for calculating the strength and dimensions vary for the several cases of each of those forms, the constants for calculating the dimensions in the case of *Stiffness* must vary accordingly, as will appear from the solution of the following problems.

PROB. XII. *To determine an expression for the side of a square beam of cast iron, to bear a given load, and resist a given deflexion; the length of the beam being given, and the straining force in the direction of the vertical diagonal.*

From the principles employed in the solution of the second problem, the constant number for the first case is found to be 600 nearly, and the constant for the first case of the first problem, deduced from the experimental numbers, is exactly 850, while that for the corresponding case of the eleventh problem is 42648; and since the expression for the deflexion of the beams, both in the first and second problems is the same, these numbers must be proportional:

that is, 850 : 600 : : 42648 : 30104.

Therefore, by restoring the other terms, and putting D for any deflexion, we have

1. *When the beam is supported at the ends, and loaded in the middle,*

$$l^3 w = 30104\, s^4 D.$$

And by proceeding in a similar manner for the other cases, we get

2. *When the beam is supported at the ends, but the load not in the middle,*

$$m^2 n^2 w = 1886\, l s^4 D.$$

3. *When the beam is supported at the ends, and loaded uniformly over the length,*

$$l^3 w = 48166\, s^4 D.$$

4. *When the beam is fixed at one end, and loaded at the other,*

$$l^3 w = \frac{1886\, s^4 D}{1+R}.$$

PROB. XIII. *To determine an expression for the diameter of a cylindric beam of cast iron, to bear a given load, and resist a given deflexion; the length of the beam being given.*

From the principles employed in the solution of the third problem, the constant number for the first case is found to be 500, and the constant for the deflexion being the same as in the two last problems, we have, by a similar operation,

850 : 500 : : 42648 : 25087. Hence, by restoring the terms, and putting D for any other deflexion, we get,

1. *When the beam is supported at the ends, and loaded in the middle,*

$$l^3 w = 25087\, d^4 \mathrm{D}.$$

2. *When the beam is supported at the ends, but the load not in the middle,*

$$m^2 n^2 w = 1571\, l d^4 \mathrm{D}.$$

3. *When the beam is supported at the ends, and loaded uniformly over the length,*

$$l^3 w = 40139\, d^4 \mathrm{D}.$$

4. *When the beam is fixed at one end, and loaded at the other,*

$$l^3 w = \frac{1571\, d^4 \mathrm{D}}{1 + \mathrm{R}}.$$

PROB. XIV. *To determine an expression for the exterior diameter of a tubular beam of cast iron, to bear a given load and resist a given deflexion ; the length of the beam being given.*

By proceeding in the same manner as we did in the solution of the last problem, we find the general expressions for the several cases of *tubular* beams to be

$$
\left.
\begin{array}{ll}
1. & l^3 w = 25087\, d^4 D(1 - p^4), \\
2. & m^2 n^2 w = 1571\, l d^4 D(1 - p^4), \\
3. & l^3 w = 40139\, d^4 D(1 - p^4), \\
4. & l^3 w = \dfrac{1571\, d^4 D(1 - p^4)}{1 + R}.
\end{array}
\right\} \quad (E)
$$

Where p is the ratio of the interior to the exterior diameter, or, as we have shewn in the solution of the fourth problem, the quotient that arises from dividing the interior by the exterior diameter of the beam. But taking the particular value of p, viz. ·7166 (which number is the diameter of the hollow part, when the exterior diameter is unity), then the expressions for the several cases of the problem will be as follow:

1. *When the beam is supported at the ends, and loaded in the middle,*

$$l^3 w = 18471\, d^4 D.$$

2. *When the beam is supported at the ends, but the load not in the middle,*

$$m^2 n^2 w = 1156\, l d^4 D.$$

3. *When the beam is supported at the ends, and loaded uniformly over the length,*

$$l^3 w = 29542\, d^4 D.$$

4. *When the beam is fixed at one end, and loaded at the other,*

$$l^3 w = \frac{1156\, d^4 D}{1 + R}.$$

PROB. XV. *To determine an expression for the dimensions of a grooved beam of cast iron, to bear a given load and resist a given deflexion; the length of the beam being given.*

By proceeding in the same manner as we did for the rectangular beam in the eleventh problem, we find the general expressions for the several cases of *grooved* beams to be

$$\left.\begin{array}{ll} 1. & l^3 w = 42648\, b\, d^3 \mathrm{D}(1-q p^3), \\ 2. & m^2 n^2 w = 2665\, l\, b\, d^3 \mathrm{D}(1-q p^3), \\ 3. & l^3 w = 68236\, b\, d^3 \mathrm{D}(1-q p^3), \\ 4. & l^3 w = \dfrac{2665\, b\, d^3\, \mathrm{D}\,(1-q p^3)}{1+\mathrm{R}}. \end{array}\right\} \quad (\mathrm{F})$$

By taking the particular values assigned to p and q in the fifth problem, viz. $p = \cdot 7$ and $q = \cdot 625$, the expressions for the several cases of *grooved* beams will be as follow:

1. *When the beam is supported at the ends, and loaded in the middle,*

$$l^3 w = 33516\, b\, d^3 \mathrm{D}.$$

2. *When the beam is supported at the ends, but the load not in the middle,*

$$m^2 n^2 w = 2099\, l\, b\, d^3 \mathrm{D}.$$

3. *When the beam is supported at the ends, and loaded uniformly over the length,*

$$l^3 w = 53625\, b\, d^3 \mathrm{D}.$$

4. *When the beam is fixed at one end, and loaded at the other,*

$$l^3w = \frac{2099\,b\,d^3\,\mathrm{D}}{1+\mathrm{R}}.$$

PROB. XVI. *To determine an expression for the dimensions of an open beam of cast iron, to bear a given load and resist a given deflexion; the length of the beam being given.*

The same principle of comparison being adopted as in the solutions of the preceding problems, the general expressions for the several cases of *open* beams will be as below:

$$\left.\begin{array}{ll} 1. & l^3w = 42648\,b\,d^3\mathrm{D}(1-p^3), \\ 2. & m^2n^2w = 2665\,l\,b\,d^3\mathrm{D}(1-p^3), \\ 3. & l^3w = 68236\,b\,d^3\mathrm{D}(1-p^3), \\ 4. & l^3w = \dfrac{2665\,b\,d^3\,\mathrm{D}\,(1-p^3)}{1+\mathrm{R}}. \end{array}\right\} \quad (\mathrm{G})$$

But taking the particular value assigned to p in the sixth and fifteenth problems, viz. $p = {\cdot}7$, the expressions for the several cases of *open* beams will be as follow:

1. *When the beam is supported at the ends, and loaded in the middle,*

$$l^3w = 28020\,b\,d^3\mathrm{D}.$$

2. *When the beam is supported at the ends, but the load not in the middle,*

$$m^2n^2w = 1760\,l\,b\,d^3\mathrm{D}.$$

3. *When the beam is supported at the ends, and loaded uniformly over the length,*

$$l^3w = 44794\, b\, d^3 \mathrm{D}.$$

4. *When the beam is fixed at one end, and loaded at the other,*

$$l^3w = \frac{1760\, b\, d^3 \mathrm{D}.}{1+\mathrm{R}}$$

For the purpose of reference and comparison, we shall collect the expressions for the several cases of the six foregoing problems into a tabular form, as under:—

Table of Formulæ for calculating the Dimensions of Cast Iron Beams, to resist a given Deflexion.

	Rectangular Beams.		Tubular Beams.
1	$l^3 w = 42648\, b\, d^3 D$	1	$l^3 w = 18471\, d^4 D$
2	$m^2 n^2 w = 2665\, l\, b\, d^3 D$	2	$m^2 n^2 w = 1156\, l\, d^4 D$
3	$l^3 w = 68236\, b\, d^3 D$	3	$l^3 w = 29542\, d^4 D$
4	$l^3 w = \frac{2665\, b\, d^3 D}{1 + R}$	4	$l^3 w = \frac{1156\, d^4 D}{1 + R}$
	Square Beams.*		**Grooved Beams.**
1	$l^3 w = 30104\, s^4 D$	1	$l^3 w = 33516\, b\, d^3 D$
2	$m^2 n^2 w = 1886\, l\, s^4 D$	2	$m^2 n^2 w = 2099\, l\, b\, d^3 D$
3	$l^3 w = 48166\, s^4 D$	3	$l^3 w = 53652\, b\, d^3 D$
4	$l^3 w = \frac{1886\, s^4 D}{1 + R}$	4	$l^3 w = \frac{2099\, b\, d^3 D}{1 + R}$
	Cylindric Beams.		**Open Beams.**
1	$l^3 w = 25087\, d^4 D$	1	$l^3 w = 28020\, b\, d^3 D$
2	$m^2 n^2 w = 1571\, l\, d^4 D$	2	$m^2 n^2 w = 1760\, l\, b\, d^3 D$
3	$l^3 w = 40139\, d^4 D$	3	$l^3 w = 44794\, b\, d^3 D$
4	$l^3 w = \frac{1571\, d^4 D}{1+R}$	4	$l^3 w = \frac{1760\, b\, d^3 D}{1 + R}$

* This class of formulæ apply to square beams, when the direction of the straining force coincides with the vertical diagonal; those that apply to square beams, when the straining force is parallel to the side, have been given in the note to Problem XI.

The forms of section exhibited in the table of formulæ at p. 18, must be understood as belonging also to the expressions tabulated above.

Such is the theory of *Stiffness* for those forms of beams which we have treated more at large in a former part of the work; but in the tenth problem we alluded to several other forms that sometimes occur, and for which we calculated the deflexions, viz.

Beams having the outline of the depth a parabola;

Beams having the outline of the depth an ellipsis;

Beams having the breadth uniform, and the depth at the ends half the depth at the middle;

Beams having the depth uniform, and the breadth bounded by a triangle.

NOTE. It is evident that the *rectangular*, the *grooved*, or I-formed, and the *open* beam, will each admit of the parabolic and elliptic forms mentioned above.

We shall, therefore, according to our general plan, give a tabular view of the formulæ for calculating the stiffness for these several forms in the most useful cases; where we have to remark, that the first expression in each compartment, corresponds to the case, *when the beam is supported at the ends, and loaded in the middle;* and the second expression to the case, *when the beam is fixed at one end, and loaded at the other.*

Table of Formulæ for calculating the Dimensions of Parabolic and Elliptic Beams, to resist a given Deflexion.

	Parabolic Rectangular.*		Elliptic Rectangular.*
1	$l^3 w = 21324\, b\, d^3 D$	1	$l^3 w = 33176\, b\, d^3 D$
2	$l^3 w = \frac{1332\, b\, d^3\, D}{1 + R}$	2	$l^3 w = \frac{2074\, b\, d^3\, D}{1 + R}$
	Parabolic Grooved.		**Elliptic Grooved.**
1	$l^3 w = 16758\, b\, d^3 D$	1	$l^3 w = 26072\, b\, d^3 D$
2	$l^3 w = \frac{1049\, b\, d^3\, D}{1 + R}$	2	$l^3 w = \frac{1630\, b\, d^3\, D}{1 + R}$
	Parabolic Open.		**Elliptic Open.**
1	$l^3 w = 14010\, b\, d^3 D$	1	$l^3 w = 21779\, b\, d^3 D$
2	$l^3 w = \frac{880\, b\, d^3\, D}{1 + R}$	2	$l^3 w = \frac{1361\, b\, d^3\, D}{1 + R}$

The general expressions for calculating the stiffness of parabolic and elliptic *grooved*, or I-formed and *open* sections, are as exhibited in the following table:

* The terms parabolic rectangular and elliptic rectangular simply mean, that the beams are uniform in thickness or breadth throughout the length, and have the outline of the upper side a parabola or an ellipsis.

General Formulæ for calculating the Dimensions of Parabolic and Elliptic Beams, to resist a given Deflexion.

	Parabolic Grooved.		*Elliptic Grooved.*
1	$l^3 w = 21324\, b\, d^3 D (1 - q p^3)$	1	$l^3 w = 33176\, b\, d^3 D (1 - q p^3)$
2	$l^3 w = \frac{1332\, b\, d^3 D (1 - q p^3)}{1 + R}$	2	$l^3 w = \frac{2074\, b\, d^3 D (1 - q p^3)}{1 + R}$
	Parabolic Open.		*Elliptic Open.*
1	$l^3 w = 21324\, b\, d^3 D (1 - p^3)$	1	$l^3 w = 33176\, b\, d^3 D (1 - p^3)$
2	$l^3 w = \frac{1332\, b\, d^3 D (1 - p^3)}{1 + R}$	2	$l^3 w = \frac{2074\, b\, d^3 D (1 - p^3)}{1 + R}$

Next, for beams having the breadth uniform, and the depth at the extremities one half the depth at the middle, we have

1. *When the beam is supported at the ends, and loaded in the middle,*

$$l^3 w = 26084\, b\, d^3 D.$$

2. *When the beam is fixed at one end, and loaded at the other,*

$$l^3 w = \frac{1630\, b\, d^3 D}{1 + R}.$$

Again, for beams having the depth uniform, and the breadth bounded by a triangle, we have

1. *When the beam is supported at the ends, and loaded in the middle.*

$$l^3 w = 28432\, b\, d^3 D.$$

2. *When the beam is fixed at one end, and loaded at the other,*

$$l^3 w = \frac{1777\, b\, d^3\, D}{1 + R}.$$

Having thus established the formulæ* from

* The only objection that can be made to the formulæ that we have derived for calculating the dimensions of beams to resist a given deflexion, is the largeness of the constant numbers, which renders the actual calculation tedious; but to persons acquainted with the use of logarithms, this objection will be of little consequence; and since the reciprocals of the constants will generally be very small, they may be used as multipliers on the other side of the equation with some advantage. The following is a table of the reciprocals for the constants in the several formulæ, arranged according to their magnitudes:—

Constants.	*Reciprocals.*	*Constants.*	*Reciprocals.*
880	·00114	21324	·000046
1049	·00095	21779	·000046
1156	·00086	25087	·00004
1332	·00075	26072	·000038
1360	·00074	26084	·000038
1361	·00074	28020	·000036
1571	·00064	28432	·000035
1630	·00061	29542	·000034
1760	·00057	30104	·000033
1777	·00056	33176	·00003
1886	·00053	33516	·00003
2074	·00048	40139	·00003
2099	·00047	42648	·000023
2665	·00037	44794	·000022
14010	·00007	48166	·000021
16758	·00006	53652	·000019
18471	·000054	68236	·000014

which the rules are deduced for calculating the stiffness of beams of various forms, and under various circumstances, we shall, in the next place, give two or three examples, to illustrate the manner in which the formulæ are to be applied; and that the reader's confidence may be confirmed in the truth of our deductions, it may be proper to select the examples from Mr. Tredgold's work on cast iron: previously, however, we shall deliver a rule for the resolution of the formulæ, which will be found useful to those not versed in algebraic reductions.

GENERAL RULE.

Substitute the given dimensions and their powers, with the given weight and deflexion, for the representatives of each in the respective formula, and another expression will arise, involving one unknown term only; then, divide both sides of this new expression by the number with which the unknown quantity is combined and such root of the quotient as is denoted by the index of the unknown letter, will give the answer.

Example 1. Let it be required to determine the depth of the beam for a pumping engine, to bear a strain of 30000 lbs. and resist a deflexion of 0·25 inches, its breadth being 5 inches, and its length 24 feet; the parts on each side of the centre of motion being equal. (*See* TREDGOLD on Cast Iron, art. 212).

If the beam be supposed uniform, both in breadth and in depth, throughout the length, it corresponds to Case 4 of Problem XI, or to No. 4 of the tabulated class for uniform rectangular beams, the formula for which is

$$l^3 w = \frac{2665\, b\, d^3\, D}{1 + R}$$

And since the parts on each side of the centre of motion are equal by the question, we have $R = 1$, and the formula becomes

$$2\, l^3 w = 2665\, b\, d^3 D.$$

Now $b = 5$ inches, $l = 12$ feet, $w = 30000$ lbs. and $D = 0{\cdot}25$ inches; therefore (by the general rule), let these numbers be substituted for their representatives in the preceding formula, and we get

$$2 \times 12^3 \times 30000 = 2665 \times 5 \times {\cdot}25 \times d^3;$$

$$\text{or, } d^3 = \frac{2 \times 12^3 \times 30000}{2665 \times 5 \times {\cdot}25} = 31123{\cdot}45.$$

Here the index of the depth is 3, consequently, by extracting the cube root, we obtain 31·4 inches for the depth of the beam.

Example 2. Let it be required to determine the diameter of a solid cylindric beam, to bear a load of 3472 lbs. in the middle, its length being 21 feet, and the deflexion not to exceed half an inch. (*See* TREDGOLD on Cast Iron, art. 220.) The conditions of the beam in this example, correspond to the first case of the thirteenth problem, or to No. 1 of the

tabulated class for cylindric beams, the formula for which is

$$l^3 w = 25087\, d^4 \mathrm{D}.$$

Now $l = 21$ feet, $w = 3472$ lbs. and $\mathrm{D} = 5$ inches; let these numbers be substituted for l, w and D, in the preceding equation, and it becomes

$$21^3 \times 3472 = 25087 \times \cdot 5 \times d^4;$$

$$\text{or, } d^4 = \frac{21^3 \times 3472}{25087 \times \cdot 5} = 2563 \cdot 4146.$$

Now the index of the diameter is 4; hence, if the fourth root of 2563·4146 be extracted, the diameter will be 7·12 inches.

Example 3. Let it be required to determine the diameter of a tubular beam, to bear a load of 3472 lbs. in the middle, its length being 21 feet, and the deflexion not to exceed half an inch; the interior being seven-tenths of the exterior diameter (*See* Tredgold on Cast Iron, art. 221). The conditions of the beam in this example, correspond to the first case of the fourteenth problem; but since the ratio of the interior to the exterior diameter, is not here the same as we have assigned it, we must take the first of the general expressions to Problem XIV. viz.

$$l^3 w = 25087\, d^4 \mathrm{D}\,(1 - p^4). \quad \text{(No. 1, Class E).}$$

Here $l = 21$ feet, $w = 3472$ lbs. $\mathrm{D} = \cdot 5$ inches, and $p = \cdot 7$; let these numbers be substituted for l, w, D and p, in the preceding expression, and it becomes

$$21^3 \times 3472 = 25087 \times \cdot 5 \times (1 - \cdot 7^4) \times d^4;$$

or, $d^4 = \dfrac{21^3 \times 3472}{25087 \times \cdot 5 \times \cdot 378} = 3361\cdot 53$; and the index of d the diameter is 4; therefore, by extracting the fourth root of 3361·53, we get 7·62 inches for the exterior diameter; and $7\cdot 62 \times \cdot 7 = 5\cdot 334$ inches for the diameter of the part to be left hollow: hence, $\dfrac{7\cdot 62 - 5\cdot 334}{2} = 1\cdot 143$ inches for the thickness of metal.

Example 4. Let it be required to determine the depth of a uniform rectangular beam, to bear a weight of 33750 lbs. equally diffused over the length, its breadth being 2 inches, length 15 feet, and the deflexion produced by the load ·46 inches. (*See* TREDGOLD on Cast Iron, arts. 115 and 179).

Here the conditions of the beam correspond to the third case of Problem XI. or No. 3 of the tabulated class for rectangular beams, the formula for which is

$l^3 w = 68236\, b\, d^3 \mathrm{D}$, which, by substituting the given numbers, becomes

$$15^3 \times 33750 = 68236 \times 2 \times \cdot 46 \times d^3;$$

or, $d^3 = \dfrac{15^3 \times 33750}{68236 \times 2 \times \cdot 46} = 1814\cdot 455$; and the index of d the depth is 3; therefore, by extracting the cube root of 1814·455, we have the depth equal to 12·2 inches nearly.

The results of the preceding examples obtained

by our formulæ, being compared with those given by Mr. Tredgold, will shew the degree of confidence to be placed in the accuracy of the principles on which our theory is founded ; and the simple arrangement given to the equations in the tabular form, will, it is presumed, render it a matter of no great difficulty, for the intelligent reader to calculate any case that may fall under his consideration.

We shall next go on to shew, in what manner the equations are to be modified, for the purpose of calculating the dimensions of beams of wrought iron, and the several sorts of wood mentioned in the following table of the relative stiffness of materials, that of cast iron being unity.

Table of the relative Stiffness of Materials, Cast Iron Unity.

Ash	·089
Beech	·073
Elm	·073
Fir, red or yellow.....	·1154
Fir, white..........	·1
Oak, English	·093
Pine, American yellow	·087
Wrought Iron	1·3

Hence, to adapt our theory to the calculation of beams of the above materials, it is only requisite to multiply the constant in the several ex-

pressions for cast iron beams, by the numbers in the table opposite the material in question, and the products will be the constants for calculating the dimensions of similar beams of the said materials, the stiffness béing the same: one example will illustrate the whole.

Let it be required to find the depth of a rectangular beam of American pine, to bear a weight of 8960 lbs. in the middle of its length, the breadth being 3 inches, length 18 feet, and the deflexion not to exceed $\frac{1}{4}$ of an inch. The formula for a cast iron beam of this form, and so loaded, is

$l^3w = 42648\,b\,d^3\,\text{D}$ (see No. 1, class for rectangular beams); therefore, $42648 \times \cdot087 = 3710\cdot376 =$ the constant for American pine, and the equation becomes, by substitution,

$$l^3w = 3710\,b\,d^3\text{D}.$$

Now $b = 3$ inches, $l = 18$ feet, $w = 8960$ lbs. and $\text{D} = \cdot25$ of an inch; therefore, by substituting these numbers for their representatives in the preceding equation, we have

$$18^3 \times 8960 = 3710 \times 3 \times \cdot25 \times d^3;$$

$$\text{or, } d^3 = \frac{18^3 \times 8960}{3710 \times 3 \times \cdot25} = 1877\cdot9.$$

Hence, by extracting the cube root of 1877·9 we get 26·58 inches for the depth of a beam of American pine, that will not be deflected more than $\frac{1}{4}$ of an inch in the middle by a load of 8960 lbs.

Numerous interesting examples might be proposed respecting the parts of machinery and other important practical subjects; but the work having already extended beyond the bounds originally designed for it, we have thought proper to omit them.

Such, then, is the theory of resistance to *Transverse Strains*; but the resistance to twisting or *torsion*, the resistance to *compression* and *tension*, and the resistance to *impulsion*, remain to be considered.

THE

THEORY OF

Mr. BRAMAH'S

HYDRO-MECHANICAL PRESS,

WHICH IS THE MOST APPROVED MACHINE FOR TRYING THE STRENGTH OF BEAMS WHEN EXPOSED TO TRANSVERSE STRAINS.

THE

HYDRO-MECHANICAL PRESS.

In order effectually to prove the strength of beams, as calculated by the principles laid down in the foregoing part of this work, it is necessary to have at hand some very powerful and commodious instrument, one that is easy of management and delicate in its indications, so that the straining force can be removed, and the beams suffered to restore themselves, when they have attained the degree of flexure intended to be produced, and which theory assigns to them.

Of all the engines now in use, the *Hydro-Mechanical Press* is the most convenient for this purpose, not only on account of its immense power, but because of the accuracy with which it indicates the instant when the desired effect has been produced. This very ingenious and powerful machine is founded on the following hydrostatical principle: viz. *When a mass of fluid, in a state of equilibrium, is subjected to the action of any forces, every particle of the fluid is pressed equally in every direction.* Therefore, if any number of pistons of different sizes are ap-

plied to apertures in the sides of a vessel full of water, the forces with which the pistons are pressed, will be *in equilibrio* if they are proportional to the areas of the pistons on which they act.

By attentively considering the above principle, the late Joseph Bramah, Esq. of Pimlico, succeeded in rendering a law of nature subservient to the most important purposes; such as working cranes, pulling up the roots of trees, trying the strength of materials, packing goods and the like. This is done by forcing an incompressible fluid through a small tube, into a cylinder of considerable strength, furnished with a solid moveable water-tight piston, which we shall designate the *ram*, merely to distinguish it from the piston of the forcing pump. Into the bottom of this cylinder the tube is inserted, and communicates with a forcing pump placed in a cistern which contains the water; the power is applied to a lever attached to the pump, and the piston, pressing on the surface of the water, communicates its force (through the intervention of the fluid) to the *ram*, or piston of the cylinder, to the top of which the work to be performed is applied.

It is the proportion between the diameter of the forcing pump and the diameter of the cylinder that constitutes the principal feature of the machine, and on which its excellence chiefly depends; for since the one can be increased and the other decreased at

pleasure, it is evident there can be no limit to its power. If the diameter of the pump and that of the cylinder be equal, a force of one pound on the piston will transmit a pressure of one pound only to the *ram;* but if the diameter of the pump be half that of the cylinder, a force of one pound on the piston will transmit a pressure of four pounds to the *ram*, and ten pounds will transmit forty; that is, *the force and the effect, are to each other directly as the square of the diameter of the pump, to the square of the diameter of the cylinder.* This is the principle, and the theory is developed in what follows.

Notation.

Put $d =$ the diameter of the forcing pump,
$f =$ the force with which the piston descends;
$\mathrm{D} =$ the diameter of the cylinder,
$\mathrm{P} =$ the pressure on the *ram;*
$\delta =$ the diameter of the safety valve,
and $p =$ the pressure thereon.

Then, the several particulars of the *Hydro-Mechanical Press* are exhibited in the following analogies:

$$d^2 : \mathrm{D}^2 :: f : \mathrm{P},$$
$$\delta^2 : \mathrm{D}^2 :: p : \mathrm{P},$$
$$d^2 : \delta^2 :: f : p,$$
$$\mathrm{D}^2 : \delta^2 :: \mathrm{P} : p.$$

And from these four analogies arise the following set of equations:

1. $P=\frac{D^2 f}{d^2}$, 2. $P=\frac{D^2 p}{\delta^2}$,

3. $p=\frac{\delta^2 f}{d^2}$, 4. $p=\frac{\delta^2 P}{D^2}$,

5. $D=\frac{d\sqrt{P}}{\sqrt{f}}$, 6. $D=\frac{\delta\sqrt{P}}{\sqrt{p}}$,

7. $\delta=\frac{d\sqrt{p}}{\sqrt{f}}$, 8. $\delta=\frac{D\sqrt{p}}{\sqrt{P}}$,

9. $d=\frac{D\sqrt{f}}{\sqrt{P}}$, 10. $d=\frac{\delta\sqrt{f}}{\sqrt{p}}$,

11. $f=\frac{d^2 P}{D^2}$, 12. $f=\frac{d^2 P}{\delta^2}$.

Moreover, if t represent the thickness of metal in the cylinder, capable of sustaining the pressure while the elastic force of the material remains perfect, then, from different principles we obtain

13. $t=\frac{\cdot 000083 f R}{d^2-\cdot 000083 f}$, where $R=\frac{1}{2}D$ the radius of the cylinder.

Again, if n be the pressure in lbs. per square inch, we have

$$14.\ n=\frac{f}{\cdot 7854\, d^2}.$$

By these fourteen equations, every particular respecting the *Hydro-Mechanical Press* can be calculated, and the method of procedure will become manifest from the following examples.

Example 1. If a force of 560 lbs. act on the piston of a forcing pump whose diameter is half an

inch, what pressure will be transmitted to the *ram*, supposing its diameter to be 12 inches?

The formula is No. 1, viz.

$$P=\frac{D^2 f}{d^2}:$$

Now, $d=\frac{1}{2}$ an inch, D = 12 inches, and $f=560$ lbs. Let these numbers be substituted for their representatives d, D and f in the equation above, and it becomes

$$P=\frac{12^2\times 560}{\cdot 5^2};$$

Whence P = 322560 lbs. or 144 tons.

Example 2. If a weight of 35 lbs. on the safety valve, whose diameter is one-eighth of an inch, be just balanced, what is the pressure on the *ram*, its diameter being 12 inches?

The formula is No. 2, viz.

$$P=\frac{D^2 p}{\delta^2}:$$

But $\delta=\frac{1}{8}$ of an inch, D = 12 inches, and $p=35$ lbs. Let these numbers be substituted for δ, D and p in the equation, and we get

$$P=\frac{12^2\times 35}{\cdot 125^2};$$

that is, P = 322560 lbs. or 144 tons, as before.

Example 3. If a force of 560 lbs. act on the piston of a forcing pump whose diameter is half an

inch, what weight will it balance on the safety valve, its diameter being one-eighth of an inch?

The formula is No. 3, viz.

$$p=\frac{\delta^2 f}{d^2}:$$

Here $d=\frac{1}{2}$ an inch, $\delta=\frac{1}{8}$ of an inch, and $f=560$ lbs. Let these numbers be substituted for d, δ and f in the equation, and we obtain

$$p=\frac{\frac{1}{64}\times 560}{\frac{1}{4}};$$

$$\text{that is, } p=\frac{560}{16}=35 \text{ lbs.}$$

Example 4. If the *ram*, whose diameter is 12 inches, sustain a pressure of 144 tons or 322560 lbs. what weight will it balance on the safety valve, its diameter being one-eighth of an inch?

The formula is No. 4, viz.

$$p=\frac{\delta^2 \mathrm{P}}{\mathrm{D}^2}:$$

Now, $\mathrm{D}=12$ inches, $\mathrm{P}=322560$ lbs. and $\delta=$ one-eighth of an inch. Let these numbers be substituted for D, P and δ, and we get

$$p=\frac{\frac{1}{64}\times 322560}{144};$$

$$\text{Wherefore } p=\frac{322560}{9216}=35 \text{ lbs. as before.}$$

Example 5. If a force of 560 lbs. act on the piston of a forcing pump whose diameter is half an

inch, what must be the diameter of the *ram* to sustain a pressure of 144 tons, or 322560 lbs.?

The formula is No. 5, viz.

$$D=\frac{d\sqrt{P}}{\sqrt{f}}.$$

Here $d=\frac{1}{2}$ an inch, $f=560$ lbs. and $P=322560$ lbs. Let these numbers be substituted for d, f and P, and our equation becomes

$$D=\tfrac{1}{2}\sqrt{\frac{322560}{560}};$$

that is, $D=\frac{1}{2}\sqrt{576}=12$ inches.

Example 6. If 35 lbs. on the safety valve, whose diameter is one-eighth of an inch, balance a pressure of 322560 lbs. on the *ram*, what is its diameter?

The formula is No. 6, viz.

$$D=\frac{\delta\sqrt{P}}{\sqrt{p}}.$$

Now, $\delta=\frac{1}{8}$ of an inch, $p=35$ lbs. and $P=322560$ lbs. Let these numbers be substituted for δ, p and P in the above equation, and it becomes

$$D=\tfrac{1}{8}\sqrt{\frac{322560}{35}};$$

that is, $D=\frac{1}{8}\sqrt{9216}=12$ inches, as before.

Example 7. If a force of 560 lbs. act on the piston of a forcing pump whose diameter is half an inch, what must be the diameter of the safety valve to require a weight of 35 lbs.?

The formula is No. 7, viz.

$$\delta=\frac{d\sqrt{p}}{\sqrt{f}}.$$

Here $d=\frac{1}{2}$ an inch, $f=560$ lbs. and $p=35$ lbs. Let these numbers be substituted in the equation, and it becomes

$$\delta=\tfrac{1}{2}\sqrt{\frac{35}{560}};$$

Wherefore $\delta=\frac{1}{2}\sqrt{\frac{1}{16}}=\frac{1}{8}$ of an inch.

Example 8. If the *ram* whose diameter is 12 inches sustains a pressure of 322560 lbs. what must be the diameter of the safety valve to require a weight of 35 lbs.?

The formula is No. 8, viz.

$$\delta=\frac{D\sqrt{p}}{\sqrt{P}}:$$

But $D=12$ inches, $P=322560$ lbs. and $p=35$ lbs. Let these numbers be substituted in the equation, and we have

$$\delta=12\sqrt{\frac{35}{322560}};$$

Whence $\delta=12\sqrt{\frac{1}{9216}}=\frac{12}{96}$, or $\frac{1}{8}$ of an inch, as before.

Example 9. If a force of 560 lbs. acting on the piston of a forcing pump, transmit a pressure of 322560 lbs. to a *ram* 12 inches in diameter, what is the diameter of the pump?

The formula is No. 9, viz.

$$d = D\sqrt{\frac{f}{P}}.$$

Here D = 12 inches, f = 560 lbs. and P = 322560 lbs. Let these numbers be substituted for D, f and P, and our equation becomes

$$d = 12\sqrt{\frac{560}{322560}};$$

$$\text{that is, } d = 12\sqrt{\frac{1}{576}} = 12 \times \tfrac{1}{24} = \tfrac{1}{2} \text{ an inch.}$$

Example 10. If a force of 560 lbs. acting on the piston of a forcing pump, transmit a pressure of 35 lbs. to the safety valve whose diameter is one-eighth of an inch, what is the diameter of the pump?

The formula is No. 10, viz.

$$d = \delta\sqrt{\frac{f}{p}}.$$

Now δ = one-eighth of an inch, f = 560 lbs. and p = 35 lbs. Let these be substituted in the equation for δ, f and p, and it becomes

$$d = \tfrac{1}{8}\sqrt{\frac{560}{35}};$$

$$\text{that is, } d = \tfrac{1}{8}\sqrt{16} = \tfrac{1}{2} \text{ an inch, as before.}$$

Example 11. What force, acting on the piston of a forcing pump of half an inch diameter, will transmit a pressure of 322560 lbs. to a *ram* whose diameter is 12 inches?

The formula is No. 11, viz.

$$f=\frac{d^2P}{D^2}.$$

But $d=\frac{1}{2}$ an inch, $D=12$ inches, and $P=322560$ lbs. Let these numbers be substituted in the equation, and it becomes

$$f=\frac{\cdot5^2\times322560}{12^2};$$

$$\text{that is, } f=\frac{322560}{576}=560\text{ lbs.}$$

Example 12. What force, acting on the piston of a forcing pump of half an inch diameter, will transmit a pressure of 35 lbs. to a safety valve whose diameter is one-eighth of an inch?

The formula is No. 12, viz.

$$f=\frac{d^2p}{\delta^2}.$$

Here $d=\frac{1}{2}$ an inch, $\delta=\frac{1}{8}$ an inch, and $p=35$ lbs. Substitute these numbers in the equation, and it becomes

$$f=\frac{\cdot5^2\times35}{\cdot125^2}$$

that is, $f=35\times16=560$ lbs. as before.

Example 13. If a force of 560 lbs. act on the piston of a forcing pump whose diameter is half an inch, what must be the thickness of metal (cast iron) in the cylinder of 6 inches radius, to sustain the pressure without injury?

The formula is No. 13, viz.

$$t=\frac{\cdot000083fR}{d^2-\cdot000083f}.$$

Now $d=\frac{1}{2}$ an inch, $R=6$ inches, and $f=560$ lbs. Let these numbers be substituted for d, R, and f, and our equation becomes

$$t=\frac{\cdot000083\times560\times6}{\cdot5^2-\cdot000083\times560};$$

that is, $t=\frac{\cdot27888}{\cdot20352}=1\cdot37$ inches nearly, the thickness sought.

Note.—Dr. Gregory, in his *Mathematics for Practical Men*, has given another formula for calculating the thickness of metal, which he ascribes to Mr. Barlow; it is as follows, viz.

$t=\frac{nR}{c-n}$: Where $n=$ the number of lbs. on a square inch, $R=$ the radius of the cylinder, and $c=15300$ lbs. (the cohesive force of cast iron). Now, to apply this formula, it is necessary that the pressure on a square inch be known, and this is determined as in the following:

Example 14. If a force of 560 lbs. act on the piston of a forcing pump whose diameter is half an inch, what is the corresponding pressure on a square inch?

The formula is No. 14, viz.

$$n=\frac{f}{\cdot7854d^2}.$$

Here $d=\frac{1}{2}$ an inch, and $f=560$ lbs. Hence, by substitution, our equation becomes

$$n=\frac{560}{\cdot 7854\times\cdot 5^2};$$

that is, $n=\frac{560}{\cdot 19635}=2852$ lbs. Wherefore, according to Dr. Gregory's formula, we have for the thickness of metal

$$t=\frac{2852\times 6}{15300-2852};$$

that is, $t=\frac{17112}{12448}=1\cdot 37$ inches, the same as before.

THE END.

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RESISTANCE TO TORSION.

RESISTANCE TO TORSION.

In the foregoing part of this work, we have considered the several modifications of beams as regards the *strength*, *flexure*, and *stiffness*, when exposed to transverse strains from pressure or weight; we have therefore, in the next place, to examine what are the conditions of beams, when the straining force operates to produce fracture by means of *twisting* or *wrenching*, technically designated Torsion.

It is demonstrated by writers on the resistance of solids, that when a beam is fixed in a wall at one end, and a load applied at the other, the strain produced by the load so applied, is similar to that which endeavours to wrench a beam asunder, by a force acting at the extremity of a lever perpendicular to its axis. This species of strain generally exerts itself on beams that are employed as shafts or axles in connecting the parts of machinery; and the forms of beams usually employed for this purpose, are the *rectangular*, the *square*, and the *cylindrical*; the last of which, when the beams are large, are sometimes

made *tubular*, in order to acquire a greater degree of stiffness with a less quantity of metal, and to diminish the pressure and friction on the gudgeons.

We have already shown (*see* Case 4. Problems I. II. III. and IV.), that when beams of the above forms are fixed at one end and loaded at the other, the equations that express the conditions of strength and dimensions, are as follow:

For the *rectangular* beam, $lw = 212\frac{1}{2}\, b\, d^2$.

For the *square* beam, $lw = 150\, s^3$.

For the *cylindrical* beam, $lw = 125\, d^3$.

For the *tubular* beam, $lw = 125\, d^3 (1 - p^4)$.*

In each of these equations, l is the length of the beam in feet; but in considering the resistance to torsion, the length is not necessary, because the resistance is the same whatever the length may be, provided it be not less than

$$\text{the} \left\{ \begin{array}{l} \text{breadth in the } \textit{rectangular} \\ \text{diagonal of section in the } \textit{square} \\ \text{diameter in the } \textit{cylindrical} \end{array} \right\} \text{beam.}$$

(*See* TREDGOLD on Cast Iron, articles 224, 225, and 227).

But although the length of the beam is not necessary under these limitations, yet we must consider the straining force as operating at the extremity

* This is the 4th case of the general expression, Problem IV. and is found by taking one-fourth of the equation (A), page 12, agreeably to the remark at page 4.

of a lever, which, in the case of shafts and axles, is usually the radius of a wheel; therefore, if the radius, at the extremity of which the power acts, be taken in feet and represented by r, we have only to substitute r in the preceding equations instead of l, and they become, in the case of torsion:

For the *rectangular* beam, $r w = 212\frac{1}{2}\, b d^2$.

For the *square* beam, $r w = 150 s^3$.

For the *cylindrical* beam, $r w = 125 d^3$.

For the *tubular* beam, $r w = 125 d^3 (1 - p^4)$.

The foregoing equations, it will be observed, apply only to cast iron; but since shafts, axles, and other beams liable to be twisted, are frequently constructed of wrought iron, we shall, for the convenience of practical men, adapt our formulæ to that material also. But this is easily done; for by taking the constant corresponding to the 4th case of each form, from the table (page 31), we have the following equations for wrought iron, viz.

For the *rectangular* beam, $r w = 238\, b d^2$.

For the *square* beam, $r w = 168 s^3$.

For the *cylindrical* beam, $r w = 140 d^3$.

For the *tubular* beam, $r w = 140 d^3 (1 - p^4)$.

The above, therefore, is the theory of *torsion*, as applied to those forms of beams in cast and wrought iron. We shall, in the next place, give a few examples, to shew the manner of applying the formulæ in practical cases; and that nothing may be left

undone, we shall determine each quantity in terms of the others, for the four forms of beams both in cast and wrought iron.

NOTE. *It is of course always to be understood, that the strain is within the elastic power of the material.*

Example 1. What force will be resisted by a rectangular shaft of cast iron, 3 inches broad and 5 inches deep, when applied at the circumference of a wheel whose radius is 4 feet?

The formula for the rectangular beam of cast iron, is

$$r\,w = 212\tfrac{1}{2}\,b\,d^2.$$

Now $b = 3$ inches, $d = 5$ inches, and $r = 4$ feet; let these numbers be substituted for b, d and r, in the above equation, and it becomes

$$4\,w = 212\tfrac{1}{2} \times 3 \times 5^2;$$

that is, $w = \frac{15937 \cdot 5}{4} = 3984\tfrac{3}{8}$ lbs. the force required.

Let the data remain; what will be the force when the shaft is of wrought iron?

The formula for wrought iron, is

$$r\,w = 238\,b\,d^2.$$

And this, by substitution becomes

$$4\,w = 238 \times 3 \times 5^2;$$

that is, $w = \frac{17850}{4} = 4462\tfrac{1}{2}$ lbs. the force required.

From which it appears, that a wrought iron shaft of the given dimensions, resists a force of $478\frac{1}{8}$ lbs. more than a shaft of cast iron.

Example 2. A cast iron shaft 6 inches square, is turned round by a wheel whose radius is 7 feet; what power will it resist?

The formula for a square beam of cast iron, is

$$r w = 150 s^3.$$

Now $r = 7$ feet and $s = 6$ inches; let these values be substituted for r and s in the equation, and we get

$$7 w = 150 \times 6^3;$$

that is, $w = \frac{32400}{7} = 4628\frac{4}{7}$ lbs. the force required.

The data remaining; what is the force when the shaft is of wrought iron?

The formula for wrought iron, is

$$r w = 168 s^3.$$

And this by substitution becomes

$$7 w = 168 \times 6^3;$$

that is, $w = \frac{36288}{7} = 5184$ lbs. the force required.

From which it appears, that a wrought iron shaft 6 inches square, resists a force of $555\frac{3}{7}$ lbs. more than a cast iron shaft of the same dimensions.

Example 3. A water-wheel of 9 feet radius, is fixed on a solid cylindrical shaft of cast iron, whose

diameter is $5\frac{1}{4}$ inches; what force will the shaft resist?

The formula for the cylindrical beam, is

$$r\,w = 125\,d^3.$$

Here $r = 9$ feet, and $d = 5\frac{1}{4}$ inches; let these numbers be substituted for d and r in the equation, and it becomes

$$9\,w = 125 \times 5{\cdot}25^3;$$

that is, $w = \frac{18087{\cdot}89}{9} = 2009{\cdot}76$ lbs. the force required.

The data remaining; what is the force when the shaft is of wrought iron?

The formula for wrought iron, is

$$r\,w = 140\,d^3,$$

and this by substitution becomes

$$9\,w = 140 \times 5{\cdot}25^3;$$

that is, $w = \frac{20258{\cdot}46}{9} = 2250{\cdot}94$ lbs. the force required.

From which it appears, that a wrought iron shaft $5\frac{1}{4}$ inches in diameter, resists a force of 241·18 lbs. more than a cast iron shaft of the same dimensions.

Example 4. The radius of the wheel being the same as in the foregoing example, suppose the shaft to be tubular, and its exterior diameter $5\frac{1}{2}$ inches; what force will it resist, the thickness of the metal being one-fifth of the exterior diameter?

It is shewn in the solution to Prob. IV. page 12, that

$$p\,d = d - 2\,t,$$

therefore by division, we get

$$p = 1 - \frac{2\,t}{d};$$

but t by the question, is one-fifth of the diameter, or 1·1; therefore

$$p = 1 - \frac{2{\cdot}2}{5{\cdot}5} = 0{\cdot}6.$$

Now the formula for the tubular beam, is

$$r\,w = 125\,d^3\,(1 - p^4).$$

Let 0·6 be substituted for p, and it becomes, for cast iron

$$r\,w = 108{\cdot}8\,d^3 \ldots.(a)$$

Here $r = 9$ feet, and $d = 5\frac{1}{2}$ inches; let these numbers be substituted for d and r, and our equation becomes

$$9\,w = 108{\cdot}8 \times 5{\cdot}5^3;$$

that is, $w = \dfrac{18101{\cdot}6}{9} = 2011{\cdot}3$ lbs. the force required.

The data remaining; what will be the force when the shaft is of wrought iron?

The formula for wrought iron, is

$$r\,w = 140\,d^3\,(1 - p^4).$$

Let 0·6 be substituted for p, and we have

$$r\,w = 122\,d^3 \ldots.(b)$$

Now $r = 9$ feet, and $d = 5\frac{1}{2}$ inches; let these numbers be substituted for d and r, and we get

$$9\,w = 122 \times 5{\cdot}5^3;$$

that is, $w = \frac{20297{\cdot}75}{9} = 2255{\cdot}3$ lbs. the force required.

In the foregoing examples, we have considered the length of the lever and the dimensions of the beam to be given; we shall, in the next place, consider the force to be resisted, and the dimensions of the beam as being known, to determine the length of the lever, at the extremity of which the force acts.

Example 5. A rectangular shaft of cast iron, 3 inches broad and 5 inches deep, is found to resist a force of $3984\frac{3}{8}$ lbs. applied to the end of a lever; what is its length?

The formula for the rectangular beam, is

$$r\,w = 212\tfrac{1}{2}\,b\,d^2.$$

Here $b = 3$ inches, $d = 5$ inches, and $w = 3984{\cdot}375$ lbs.; let these numbers be substituted in the equation for b, d, and w, and we get

$$3984{\cdot}375\,r = 212{\cdot}5 \times 3 \times 5^2;$$

that is, $r = \frac{15937{\cdot}5}{3984{\cdot}375} = 4$ feet, for the length of the lever, or the radius of the wheel required.

The data remaining; what is the length of the lever when the shaft is of wrought iron?

The formula for wrought iron, is

$$r\,w = 238\,b\,d^2.$$

Now $b = 3$ inches, $d = 5$ inches, and $w = 3984{\cdot}375$ lbs.; let these values be substituted in the equation, and it becomes

$$3984{\cdot}375\,r = 238 \times 3 \times 5^2;$$

that is, $r = \dfrac{17850}{3984{\cdot}375} = 4{\cdot}48$ feet, for the length of the lever, or radius of the wheel required.

Example 6. A cast iron shaft 6 inches square, is found to resist a force of $4628\frac{4}{7}$ lbs. acting at the circumference of a wheel fixed thereon; what is the radius of the wheel?

The formula for a square shaft, is

$$r\,w = 150\,s^3.$$

Here $s = 6$ inches, and $w = 4628\frac{4}{7}$ lbs.; let these values be substituted for s and w in the equation, and we obtain

$$4628\tfrac{4}{7}\,r = 150 \times 6^3;$$

that is, $r = \dfrac{32400}{4628\frac{4}{7}} = 7$ feet, the radius required.

The data remaining; what is the radius when the shaft is of wrought iron?

The formula for wrought iron, is

$$r\,w = 168\,s^3.$$

Now $s = 6$ inches, and $w = 4628\frac{4}{7}$ lbs.; let these numbers be substituted for s and w in the equation, and it becomes

$$4628\tfrac{4}{7}\,r = 168 \times 6^3;$$

that is, $r = \dfrac{36288}{4628\frac{4}{7}} = 7{\cdot}84$ feet, the radius required.

Example 7. A solid cylindrical shaft of cast iron, whose diameter is $5\tfrac{1}{4}$ inches, is found to resist a force of 2009·76 lbs. applied at the circumference of a wheel fixed thereon; what is the radius of the wheel?

The formula for a cylindrical beam, is

$$r\,w = 125\,d^3.$$

Here $d = 5\tfrac{1}{4}$ inches, and $w = 2009{\cdot}76$ lbs.; let these values be substituted for d and w in the equation, and it becomes

$$2009{\cdot}76\,r = 125 \times 5{\cdot}25^3;$$

that is, $r = \dfrac{18087{\cdot}89}{2009{\cdot}76} = 9$ feet, the radius required.

The data remaining; what is the radius when the shaft is of wrought iron?

The formula for wrought iron, is

$$r\,w = 140\,d^3.$$

Now $d = 5\tfrac{1}{4}$ inches, and $w = 2009{\cdot}76$ lbs.; let these values of d and w be substituted for them in the equation, and it becomes

$$2009{\cdot}76\,r = 140 \times 525^3;$$

that is, $r = \dfrac{20258{\cdot}46}{2009{\cdot}76} = 10{\cdot}08$ feet, the radius required.

Example 8. A tubular shaft of cast iron, whose diameter is $5\frac{1}{2}$ inches, is found to resist a force of 2011·3 lbs. applied at the circumference of a wheel fixed thereon; what is the radius of the wheel, supposing the thickness of the metal to be one-fifth of the diameter?

The formula for a tubular beam in this case, is

$$r w = 108{\cdot}8 d^3.$$

(*See* equation (*a*) preceding).

Now $d = 5\frac{1}{2}$ inches, and $w = 2011{\cdot}3$ lbs.; let these values of d and w be substituted instead of them in the equation, and it becomes

$$2011{\cdot}3\, r = 108{\cdot}8 \times 5{\cdot}5^3;$$

that is, $r = \frac{18101{\cdot}6}{2011{\cdot}3} = 9$ feet, the radius required.

The data remaining; what is the radius when the shaft is of wrought iron?

The formula for wrought iron, is

$$r w = 122\, d^3.$$

(*See* equation (*b*) preceding).

Here $d = 5\frac{1}{2}$ inches, and $w = 2011{\cdot}3$ lbs.; let these values of d and w be substituted instead of them in the equation, and it becomes

$$2011{\cdot}3\, r = 122 \times 5{\cdot}5^3;$$

that is, $r = \frac{20297{\cdot}75}{2011{\cdot}3} = 10{\cdot}09$ feet, the radius required.

In the next place, we shall endeavour to ascer-

tain the dimensions of the shaft or beam, the load and the leverage being given. This is perhaps the most useful case in practice, and that which Mr. Tredgold has illustrated by an example of a cylindrical shaft for a water-wheel. (*See* his "Essay on Cast Iron," Art. 228).

Example 9. A cast iron shaft 5 inches deep, is found to resist a force of $3984\frac{3}{8}$ lbs. acting at the circumference of a wheel whose radius is 4 feet; what is the breadth of the shaft?

The formula for cast iron, is

$$r\,w = 212\tfrac{1}{2}\, b\, d^2.$$

But $d = 5$ inches, $w = 3984\frac{3}{8}$ lbs. and $r = 4$ feet; Let these numbers be substituted for d, w and r in the equation, and it becomes

$$4 \times 3984\tfrac{3}{8} = 212\tfrac{1}{2} \times 5^2 \times b;$$

$$\text{that is, } 5312{\cdot}5\, b = 15937{\cdot}5,$$

$$\text{or, } b = \frac{15937{\cdot}5}{5312{\cdot}5} = 3 \text{ inches, the}$$

breadth required.

Let the load and the leverage be the same, and suppose the breadth of the beam to be 3 inches; what is the depth?

These values of b, w and r being substituted in the equation, it becomes

$$4 \times 3984\tfrac{3}{8} = 212\tfrac{1}{2} \times 3 \times d^2;$$

$$\text{that is, } d^2 = \frac{15937{\cdot}5}{637{\cdot}5} = 25,$$

$$\text{or, } d = 5 \text{ inches, the depth re-}$$

quired.

The data in both cases remaining; determine the same for a shaft of wrought iron.

The formula for wrought iron, is

$$r\,w = 238\,b\,d^2.$$

Then, to find the breadth, we have by substitution

$$4 \times 3984\tfrac{3}{8} = 238 \times 5^2 \times b;$$

$$\text{that is, } 5950\,b = 15937{\cdot}5,$$

$$\text{or, } b = \frac{15937{\cdot}5}{5950} = 2{\cdot}68 \text{ inches, the}$$

breadth required.

To find the depth, we have by substitution

$$4 \times 3984\tfrac{3}{8} = 238 \times 3 \times d^2;$$

$$\text{that is, } d^2 = \frac{15937{\cdot}5}{714} = 22{\cdot}3214,$$

$$\text{or, } d = \sqrt{22{\cdot}3214} = 4{\cdot}72 \text{ inches,}$$

the depth required.

Example 10. A square shaft of cast iron is found to resist a load of $4628\frac{4}{7}$ lbs. acting at the circumference of a wheel whose radius is 7 feet; what is the side of the shaft?

The formula for a square beam, is

$$r\,w = 150\,s^3.$$

Here $w = 4628\frac{4}{7}$ lbs. and $r = 7$ feet, hence by substitution, our equation becomes

$$150\,s^3 = 32400;$$

$$\text{that is, } s^3 = \frac{32400}{150} = 216,$$

$$\text{or, } s = \sqrt[3]{216} = 6 \text{ inches, the side}$$

of the square required.

The data remaining; let the same be determined for a shaft of wrought iron.

The formula for wrought iron, is

$$r w = 168\, s^3.$$

And this, by substituting the above numbers, becomes

$$168\, s^3 = 32400;$$

$$\text{that is, } s^3 = \frac{32400}{168} = 192{\cdot}857,$$

$$\text{or, } s = \sqrt[3]{192{\cdot}857} = 5{\cdot}78 \text{ inches,}$$

the side of the square required.

Example 11. A cylindrical shaft of cast iron, is found to resist a force of 2000 lbs. acting at the circumference of a wheel whose radius is 9 feet; what is the diameter of the shaft?

The formula for a cylindrical beam, is

$$r w = 125\, d^3.$$

And this, by substitution becomes

$$125\, d^3 = 18000;$$

$$\text{that is, } d^3 = \frac{18000}{125} = 144,$$

$$\text{or, } d = \sqrt[3]{144} = 5{\cdot}24 \text{ inches, the}$$

diameter required.

The data remaining; let the same be found when the shaft is of wrought iron.

The formula for wrought iron, is

$$r w = 140\, d^3.$$

And this, by substitution becomes

$$140\, d^3 = 18000;$$

$$\text{that is, } d^3 = \frac{18000}{140} = 128 \cdot 57,$$

$$\text{or, } d = \sqrt[3]{128 \cdot 57} = 5 \cdot 047 \text{ inches,}$$

the diameter required.

NOTE. *The above example is the same as that given by Mr. Tredgold at art.* 228 *of his Treatise on Cast Iron; and the coincidence of the results is a proof, if any were wanted, that the theory which we have established is correct.*

Example 12. Let the load and the leverage be the same as in the preceding example, and suppose the shaft to be a hollow cylinder or tube, in which the thickness of metal is one-fifth of the exterior diameter; what is the diameter?

The formula for the tubular beam in this case, is

$$r\, w = 108 \cdot 8\, d^3. \quad (\textit{See } \text{equation } a).$$

Which becomes by substitution

$$108 \cdot 8\, d^3 = 18000;$$

$$\text{that is, } d^3 = \frac{18000}{108 \cdot 8} = 165 \cdot 438,$$

$$\text{or, } d = \sqrt{165 \cdot 438} = 5 \cdot 49 \text{ inches,}$$

the diameter required.

The data remaining; let the same be found for a wrought iron shaft.

The formula for wrought iron, is

$$r\, w = 122\, d^3. \quad (\textit{See } \text{equation } b).$$

And this by substitution, becomes

$$122\,d^3 = 18000;$$

$$\text{that is, } d^3 = \frac{18000}{122} = 131{\cdot}147,$$

$$\text{or, } d = \sqrt[3]{131{\cdot}147} = 5{\cdot}28 \text{ inches,}$$

the diameter required.

Having as we proposed, shewn the method of determining each quantity in terms of the others, we may here remark, that when the length of the shaft or axle is greater than one-eighth of the wheel's diameter, or one-fourth of the lever, the lateral stress on the shaft will always be greater than the twisting force: yet, what we have previously laid down embraces the consideration of the twisting strain only; but in shafts of great lengths in respect to their diameters, the effect of flexure must be considered; and this, of course, is the next subject that claims our attention.

It is affirmed by writers on the resistance of solids, that when a beam or shaft, whose length is greater than its diameter, is subjected to the action of any force which endeavours to twist it round, the line of greatest strain is in the direction of the diagonal of a square; and if a square be drawn on the surface of a beam in its natural form, it will become a rhombus by the action of the straining force (*see* TREDGOLD on Cast Iron, art. 229 *v*); hence it follows, that the quantity of angular motion is double

the extension of the length of the beam, therefore we get

$$7\,d\,\varphi = 16\,l,^{*} \ldots . (c)$$

where l is the length of the beam in feet, d its diameter in inches, and φ the angle of torsion.

It is shewn in the table at page 48, that the relative extensibility of wrought iron, compared to cast iron as unity, is 0·86; hence, the preceding equation becomes, for wrought iron

$$7\,d\,\varphi = 13{\cdot}76\,l \ldots . (d)$$

These two equations are sufficient for computing the angle of torsion, whatever the form of the beam or shaft may be, as is manifest from the circumstance, that no other dimensions enter the equations besides the length l and diameter d; where d may represent either the breadth of a *rectangular* beam, the side of a *square* beam, or the diameter of a *cylindrical* beam, l representing the length in them all.

Here follow a few examples to shew the method of reducing the equations.

* This equation is found in the following manner: $24\,l =$ the length of the beam in inches, of which the diameter is d inches; $\frac{1}{1204}$ is the extension in length of cast iron, and ·0174533 is the length of an arc of one degree to radius 1; therefore we get

$$\frac{24\,l}{1204} = \frac{{\cdot}0174533\,d\varphi}{2},$$ where φ is the angle of torsion; and this reduces to $7d\phi = 16\,l$, the same as we have employed above.

Example 13. The upright shaft of a mill is 30 feet long, and its diameter 10 inches; how far is it twisted when strained to the extent of its elastic force?

Here $d = 10$ inches, and $l = 30$ feet; let these numbers be substituted for d and l in equation (c) preceding, and it becomes

$$7 \times 10 \times \phi = 16 \times 30;$$

that is, $\phi = \frac{48}{7} = 6\frac{6}{7}$ degrees, for the angle of torsion. But the length of an arc of $6\frac{6}{7}$ degrees, in a circle whose radius is 5 inches, as in the present instance, is 0·6 inches nearly; hence we infer, that the strained point of the shaft, moves over something more than half an inch, before its elastic force becomes impaired.

The data remaining; determine the same for a shaft of wrought iron.

Substitute the given length and diameter in equation (d), and it becomes

$$7 \times 10 \times \phi = 13{\cdot}76 \times 30;$$

that is, $\phi = \frac{412{\cdot}8}{70} = 5{\cdot}9$ degrees nearly, for the angle of torsion in a wrought iron beam.

Example 14. A cast iron beam, 20 feet long and 6 inches the side of the square, is strained to the extent of its elastic force; over what space does the strained point move?

Substitute the given numbers 6 and 20, for d and l respectively in equation (c), and we get

$$7 \times 6 \times \phi = 16 \times 20;$$

that is, $\phi = \frac{320}{42} = 7\frac{13}{21}$ degrees, the angle of torsion, the length of which to radius $3\sqrt{2}$ is 0·56 inches nearly, the space required.

The data remaining; let the same be determined for a shaft of wrought iron. By substitution, equation (d) becomes

$$7 \times 6 \times \phi = 13{\cdot}76 \times 20;$$

that is, $\phi = \frac{275{\cdot}2}{42} = 5{\cdot}55$ degrees, the angle of torsion, the length of which to radius $3\sqrt{2}$ is 0·48 inches nearly, the space required.

Example 15. A cylindrical shaft of cast iron, 18 feet long, in being strained to the extent of its elastic force, is twisted through an angle of $6\frac{1}{2}$ degrees; what is its diameter?

Here $\phi = 6\frac{1}{2}$ degrees, and $l = 18$ feet; let these quantities be substituted in equation (c), and it becomes

$$7 \times 6\tfrac{1}{2} \times d = 16 \times 18;$$

that is, $d = \frac{288}{45{\cdot}5} = 6{\cdot}33$ inches nearly, the diameter required.

The same determined for wrought iron, is

$$d = \frac{247{\cdot}68}{45{\cdot}5} = 5{\cdot}44 \text{ inches.}$$

Example 16. A cast iron shaft, of 6 inches

diameter, in being strained to the extent of its elastic force, is twisted through an angle of 5 degrees; what is its length?

Here we have from equation (c),

$$16\,l = 7 \times 6 \times 5;$$

that is, $l = \frac{210}{16} = 13\frac{1}{8}$ feet, the length required.

For wrought iron, it is

$$l = \frac{210}{13\cdot76} = 15\cdot26 \text{ feet.}$$

So much then for the flexure of beams when exposed to a twisting strain; we have therefore, in the next place, to inquire into their stiffness to resist it. It is shewn by writers on the resistance of solids, *that the angle of torsion is directly as the straining force;* and we find from equation (c) preceding, that the expression for the angle of torsion in terms of the dimensions of the beam, is

$$\varphi = \frac{16\,l}{7\,d}$$

Hence, by comparing this equation, with those given for the strength of the *rectangular*, the *square*, the *cylindrical*, and the *tubular* beams, we shall have as follows:

For the rectangular beam.

Let b be substituted for the breadth of the beam in equation (c), and we get,

$$\frac{16l}{7b} : \varphi :: \frac{425\,b\,d^2}{2\,r} : w = \frac{2975\,b^2\,d^2\,\varphi}{32\,l\,r}.$$

For the square beam.

Let s be substituted for the side of the square in equation (c), and we get

$$\frac{16l}{7s} : \varphi :: \frac{150\,s^3}{r} : w = \frac{525\,s^4\varphi}{8\,l\,r}.$$

For the cylindrical beam.

Let d retain its place and value in equation (c), and we get

$$\frac{16l}{7d} : \varphi :: \frac{1{\cdot}25\,d^3}{r} : w = \frac{875\,d^4\varphi}{16lr}.$$

For the tubular beam.

Let d remain as in the last case, and we get

$$\frac{16l}{7\,d} : \varphi :: \frac{125d^3(1-p^4)}{r} : w = \frac{875\,d^4\varphi\,(1-p^4)}{16\,l\,r}.$$

The results of the preceding analogies, in a simplified form, are as under, viz.

For the *rectangular* beam, $l\,r\,w = 93\;b^2d^2\varphi$.

For the *square* beam, $l\,r\,w = 66\;s^4\varphi$.

For the *cylindrical* beam, $l\,r\,w = 55\,d^4\varphi$.

For the *tubular* beam, $l\,r\,w = 55\;d^4\varphi\,(1-p^4)$.

And these equations adapted to wrought iron beams, are respectively as below, viz.

For the *rectangular* beam, $l\,r\,w = 121\;b^2d^2\varphi$.

For the *square* beam, $l\,r\,w = \;86\,s^4\varphi$.

For the *cylindrical* beam, $l\,r\,w = \;72\,d^4\varphi$.

For the *tubular* beam, $l\,r\,w = \;72\,d^4\varphi\,(1-p^4)$.

These equations for wrought iron, are derived from those for cast iron, by merely multiplying each of the co-efficients into 1·3 the number in the table page 69, which denotes the relative stiffness of

wrought iron, when that of cast iron is unity. *We have given the results in round numbers.*

Having established the theory of the *strength, flexure,* and *stiffness* of beams when exposed to a twisting strain, and illustrated the method of reducing the formulæ, for every variety in the case of *strength* and *flexure,* it only now remains, to apply the formulæ for *stiffness* to a few practical examples, such as most frequently occur in the practice of the engineer; previous to which, it may not be irrelevant to remark, that in so far as practice is concerned, there will always be given, the power, the lever or radius of the wheel, the length of the shaft, and the flexure; consequently, the remaining part or parts can easily be found, as will appear from what follows.

Example 17. Let it be required to determine a rectangular shaft of cast iron 18 feet long, that shall be capable of transmitting a power equal to 2520 lbs. acting at the circumference of a wheel whose radius is 5 feet, the flexure, or twist not to exceed 4 degrees?

The formula for the rectangular beam is

$$l r w = 93\, b^2 d^2 \varphi.$$

Substitute the given numbers in this equation, and it becomes

$$18 \times 5 \times 2520 = 93 \times 4 \times b^2 d^2;$$

$$\text{that is, } b^2 d^2 = \frac{226800}{372} = 609{\cdot}6774,$$

$$\text{or } b\, d = \sqrt{609{\cdot}6774} = 24{\cdot}69 \text{ inches,}$$

for the area of section; therefore, if we assume the breadth to be $4\frac{3}{4}$ inches, we shall have the depth 5·2 inches very nearly; hence the dimensions are known.

The data remaining; determine the same when the shaft is of wrought iron.

The formula for wrought iron, is

$$l r w = 121\, b^2 d^2 \phi.$$

Substitute the given numbers in this equation, and it becomes

$$18 \times 5 \times 2520 = 121 \times 4 \times b^2 d^2;$$

$$\text{that is, } b^2 d^2 = \frac{226800}{484} = 468{\cdot}595,$$

$$\text{or, } b\, d = \sqrt{468{\cdot}595} = 21{\cdot}64 \text{ inches,}$$

the area of section. Assume the breadth $= 4{\cdot}75$ inches as before, and we shall have 4·65 inches for the depth or other dimension.

Example 18. Let it be required to find the side of a square shaft of cast iron, 12 feet long, to transmit a power of 800 lbs. acting on a pinion of $1\frac{1}{2}$ feet radius, the flexure, or angle of twist being $2\frac{1}{2}$ degrees.

The formula for a square beam, is

$$l r w = 66\, s^4 \phi.$$

Substitute the given numbers in this equation, and it becomes

$$12 \times 1\tfrac{1}{2} \times 800 = 66 \times 2\tfrac{1}{2} \times s^4;$$

$$\text{that is, } s^4 = \frac{14400}{165} = 87{\cdot}\dot{7}\dot{2},$$

$$\text{or, } s = \sqrt[4]{87{\cdot}\dot{7}\dot{2}} = 3{\cdot}05 \text{ inches,}$$

the side of the square required.

The data remaining; determine the same when the shaft is of wrought iron.

The formula for wrought iron, is

$$l r w = 86 s^4 \varphi.$$

Substitute the given numbers in this equation, and it becomes

$$12 \times 1\tfrac{1}{4} \times 800 = 86 \times 2\tfrac{1}{2} \times s^4;$$

$$\text{that is, } s^4 = \frac{14400}{215} = 66{\cdot}976744,$$

$$\text{or, } s = \sqrt[4]{66{\cdot}976744} = 2{\cdot}86 \text{ inches},$$

the side of the square required.

Example 19. Let it be required to determine the diameter of a cylindrical shaft of cast iron, 26 feet long, to transmit a power of 2000 lbs. acting at the circumference of a wheel whose radius is 9 feet, the flexure, or twisting not to exceed half a degree.

The formula for the cylindrical beam, is

$$l r w = 55 d^4 \varphi.$$

Substitute the given numbers in this equation, and it becomes

$$26 \times 9 \times 2000 = 55 \times {\cdot}5 \times d^4;$$

$$\text{that is, } d^4 = \frac{468000}{27{\cdot}5} = 17018{\cdot}\dot{1}\dot{8},$$

$$\text{or, } d = \sqrt[4]{17018{\cdot}\dot{1}\dot{8}} = 11{\cdot}42 \text{ inches},$$

the diameter required.

The data remaining; determine the same when the shaft is of wrought iron.

The formula for wrought iron is,

$$l r w = 72 d^4 \varphi.$$

Substitute the given numbers in this equation, and it becomes

$$26 \times 9 \times 2000 = 72 \times \cdot 5 \times d^4;$$

$$\text{that is, } d^4 = \frac{468000}{36} = 13000,$$

$$\text{or, } d = \sqrt[4]{13000} = 10\cdot67 \text{ inches,}$$

the diameter sought.

Example 20. Given the same as in the last example, with the shaft tubular, and the thickness of metal one-fifth of the exterior diameter; to find the diameter.

The formula for a tubular beam, is

$$l\,r\,w = 55\, d^4 \varphi\,(1 - p^4).$$

But we have shewn elsewhere, that when the thickness of metal is one-fifth of the exterior diameter, $p = 0\cdot6$ (*see* the solution of the fourth example); therefore, if $0\cdot6$ be substituted for p in the equation above, it becomes

$$l\,r\,w = 48\, d^4 \varphi.$$

Substitute the given numbers in this equation, and we get

$$26 \times 9 \times 2000 = 48 \times \cdot 5 \times d^4;$$

$$\text{that is, } d^4 = \frac{468000}{24} = 19500,$$

$$\text{or, } d = \sqrt[4]{19500} = 11\cdot81 \text{ inches,}$$

the diameter sought.

The data remaining; determine the same when the shaft is of wrought iron.

The formula for wrought iron, reduced as above, is

$$l\,r\,w = 62\, d^4 \varphi.$$

Substitute the given numbers in this equation, and it becomes

$$26 \times 9 \times 2000 = 62 \times \cdot 5 \times d^4;$$

$$\text{that is, } d^4 = \frac{468000}{31} = 15096 \cdot 7742,$$

$$\text{or, } d = \sqrt[4]{15096 \cdot 7742} = 11 \cdot 08 \text{ inches,}$$

the diameter sought.

Then for cast iron, $11 \cdot 81 \times 0 \cdot 6 = 7 \cdot 086$ inches, for the diameter of the hollow part; and for wrought iron, $11 \cdot 08 \times 0 \cdot 6 = 6 \cdot 648$ inches.

This is enough; the theory of *the resistance to torsion* is completed; and we have further to redeem our pledge, by treating in like manner of the *resistance to compression, tension,* and *impulsion,* to which object the following pages are devoted.

RESISTANCE

TO

COMPRESSION AND TENSION.

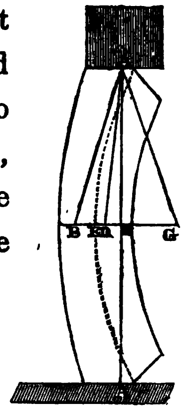
RESISTANCE

TO

COMPRESSION AND TENSION.

It is a principle in the theory of resistance, that while *the strain is within the elastic power of the material, bodies resist compression and extension with equal forces*. (*See* Tredgold on Cast Iron, art. 71). Hence it follows, that the rules which apply to the several conditions of a *compressing strain*, are equally applicable to similar conditions of an *extending one;* and for this reason, we have thought proper to rank the resistance to *compression*, and the resistance to *extension*, under one and the same head.

In establishing the theory of the *resistance to compression and tension*, we shall, for the purpose of reference, adopt the diagram employed by Mr. Tredgold; but as geometrical demonstration is entirely foreign to our plan, the reader who may feel desirous of tracing the investigation, must consult the above-named author's valuable essay on cast iron, where he will have his wishes fully gratified.



Let A A′ be a column, supported at A′, and supporting a load at A; and suppose the column to be strained to the full extent of its elastic force; then, let B G be a line in the plane of the section for which the calculations are made, and

Put l = A A′ the length of the column in feet;

b = the breadth, and

d = the depth, both in inches;

D = E G, the distance between the direction of the straining force, and the centre of the column;

φ = A G B, the angle which the direction of the straining force makes with the section of fracture;

and 15300 = the cohesive force of cast iron. (*See* TREDGOLD on Cast Iron, art. 106).

Then, by the principles of mechanics, we have

$$w = \frac{15300\, b\, d^2 \operatorname{cosec} \varphi}{d + 6\mathrm{D} + 3\, l \cot \varphi} \cdots\cdots (e)$$

And from this equation, we propose to trace the relation subsisting between the strain, and the several directions of the straining force, as follows:

1. *When the direction of the straining force coincides with the axis of the column.*

In this case, $D = o$, and $\varphi = 90°$; therefore, cosec $\varphi = 1$, and cot $\varphi = o$. Let these values of D, cosec φ, and cot φ, be substituted for them in equation (e), and it becomes

$$w = 15300\, b\, d.$$

2. *When the direction of the straining force coincides with the surface of the column.*

In this case also $\varphi = 90°$; consequently cosec $\varphi = 1$, and cot $\varphi = o$, the same as before; but $D = \frac{1}{2} d$: hence by substitution, equation (e) becomes

$$w = 3825\, b\, d.$$

3. *When the direction of the straining force is perpendicular to the plane of fracture, but not coincident with the axis or surface of the column.*

Here again $\varphi = 90°$; therefore cosec $\varphi = 1$, and cot $\varphi = o$: hence by substitution, equation (e) becomes

$$w = \frac{15300\, b\, d^2}{d + 6\,D}.$$

These three equations, it is evident, apply to the calculation of rectangular posts or columns, when their length is inconsiderable, or not more than ten or twelve times the least dimension; and when the posts are square, they become respectively

1. $w = 15300 s^2$,
2. $w = 3825 s^2$,
3. $w = \frac{15300\, s^3}{s + 6\,D}$.

And when adapted to wrought iron, they become

For the rectangular form.

1. $w = 17800\,b\,d$,
2. $w = \ 4450\,b\,d$,
3. $w = \dfrac{17800\,b\,d^2}{d+6\,\mathrm{D}}$.

For the square form.

1. $w = 17800\,s^2$,
2. $w = \ 4450\,s^2$,
3. $w = \dfrac{17800\,s^3}{s+6\,\mathrm{D}}$.

The following examples will shew the method of applying these formulæ, according as the columns may be rectangular or square, and of cast or wrought iron.

Example 1. What must be the breadth and depth of a rectangular column of cast iron, to support a load of 40568 lbs. its length being inconsiderable, and the direction of the straining force coinciding with the axis?

The formula for this case of a rectangular column, is

$$w = 15300\,b\,d;$$

but $w = 40568$ lbs. by the question; therefore, we get

$b\,d = \dfrac{40568}{15300} = 2{\cdot}65$ inches, for the area of the section; and if $d = 1\frac{1}{2}$ inches, then $b = 1{\cdot}76$ inches.*

* In the case of rectangular columns, the thickness or depth d is always taken as the least dimension.

The data remaining; determine the same for a square beam.

The formula for a square beam, is

$$w = 15300\, s^2.$$

And $w = 40568$ lbs. by the question; hence we have

$$s^2 = \frac{40568}{15300} = 2{\cdot}65;$$

or, $s = \sqrt{2{\cdot}65} = 1{\cdot}62$ inches, for the side of the square required.

Find the same things in the case of wrought iron.

The formula for wrought iron, is

$$w = 17800\, b\, d;$$

and by the question $w = 40568$ lbs. hence we have

$b\, d = \frac{40568}{17800} = 2{\cdot}28$ inches, the area of section; and if $d = 1\frac{1}{2}$ inches, then $b = 1{\cdot}52$ inches nearly.

The data remaining; determine the same for a square beam.

The formula for a square, is

$$w = 17800\, s^2;$$

but $w = 40568$ lbs. Hence we have

$$s^2 = \frac{40568}{17800} = 2{\cdot}28;$$

or, $s = \sqrt{2{\cdot}28} = 1{\cdot}51$ inches, for the side of the square required.

Example 2. The breadth of a rectangular support of cast iron is 8 inches; what must be its depth to bear a weight of 60 tons, or 134400 lbs. the direction of the force being in the surface of the column?

The formula for this case of a rectangular column, is

$$w = 3825\,bd.$$

Now $w = 13400$ lbs. and $b = 8$ inches by the question; therefore, we have

$$d = \frac{134400}{3825 \times 8} = 4{\cdot}39 \text{ inches, for the}$$

depth required.

Determine the side of a square column to support 134400 lbs.

The formula for a square, is

$$w = 3825\,s^2.$$

Therefore, we have

$$s^2 = \frac{134400}{3825} = 35{\cdot}137\,;$$

$$\text{or, } s = \sqrt{35{\cdot}137} = 5{\cdot}92 \text{ inches, for}$$

the side of the square sought.

Let the same be found when the column is of wrought iron.

The formula for wrought iron, is

$$w = 4450\,bd.$$

And by the question, $w = 134400$ lbs. and $b = 8$ inches; let these be substituted in the equation, and it becomes

$$d = \frac{134400}{4450 \times 8} = 3{\cdot}76 \text{ inches, for the}$$

depth required.

Determine the same for a square beam.

The formula for a square, is

$$w = 4450\,s^2.$$

Now $w = 134400$ lbs. by the question; therefore, we have

$$s^2 = \frac{134400}{4450} = 30{\cdot}202,$$

or, $s = \sqrt{30{\cdot}202} = 5{\cdot}49$ inches, the side of the square required.

Example 3. What must be the breadth of a rectangular column or block of cast iron, to resist a pressure of 25 tons, or 56000 lbs. supposing the thickness or depth to be 3 inches, and the direction of the straining force to meet the plane of fracture perpendicularly, at the distance of one inch from the axis of the column?

The formula for this case of the rectangular column, is

$$w = \frac{15300\, b d^2}{d + 6\,\mathrm{D}}.$$

Here we have $w = 56000$ lbs. $d = 3$ inches, and $\mathrm{D} = 1$ inch. Hence, by substitution we get

$b = \dfrac{56000 \times 9}{15300 \times 9} = 3{\cdot}66$ inches, the breadth required.

The weight and distance remaining; what is the side of the square column?

The formula for the square, is

$$w = \frac{15300\, s^3}{s + 6\,\mathrm{D}}.$$

By substituting the given quantities in this equation, we obtain after proper reduction,

$s^3 - \dfrac{560}{153} s = \dfrac{1120}{51}$, which being

reduced by the rules for the reduction of cubic equations, gives

$s=3\cdot23$ inches nearly, for the side of the square.

Determine the same for a column of wrought iron.

The formula for wrought iron, is

$$w=\frac{17800\,b\,d^2}{d+6\,D}.$$

Now $w=56000$ lbs. $d=3$ inches, and $D=1$ inch; therefore, by substitution we get

$b=\frac{56000\times 9}{17800\times 9}=3\cdot146$ inches, the breadth required.

The load and distance remaining; what is the side of the square column?

The formula for the square, is

$$w=\frac{17800\,s^3}{s+6\,D}.$$

Which, by substitution and reduction becomes

$s^3-\frac{280}{89}s=\frac{1680}{89}$, which being reduced by the rules for the reduction of cubic equations, gives

$s=3\cdot05$ inches nearly, the side of the square required.

So much for rectangular and square blocks of cast and wrought iron; but the same formulæ are readily adapted to several species of wood by means of the following tablet.

Table of Formulæ for several kinds of Wood.

Name of the Wood.	*Rectangular Columns.* Case 1.	Case 2.	Case 3.	*Square Columns.* Case 1.	Case 2.	Case 3.
Ash	$w = 3540\,bd$	$w = 885\,bd$	$w = \frac{3540\,bd^2}{d+6D}$	$w = 3540\,s^2$	$w = 885\,s^2$	$w = \frac{3540\,s^3}{s+6D}$
Beech	$w = 2360\,bd$	$w = 590\,bd$	$w = \frac{2360\,bd^2}{d+6D}$	$w = 2360\,s^2$	$w = 590\,s^2$	$w = \frac{2360\,s^3}{s+6D}$
Elm	$w = 3240\,bd$	$w = 810\,bd$	$w = \frac{3240\,bd^2}{d+6D}$	$w = 3240\,s^2$	$w = 810\,s^2$	$w = \frac{3240\,s^3}{s+6D}$
Fir, red or yellow	$w = 4290\,bd$	$w = 1073\,bd$	$w = \frac{4290\,bd^2}{d+6D}$	$w = 4290\,s^2$	$w = 1073\,s^2$	$w = \frac{4290\,s^3}{s+6D}$
Fir, white	$w = 3630\,bd$	$w = 908\,bd$	$w = \frac{3630\,bd^2}{d+6D}$	$w = 3630\,s^2$	$w = 908\,s^2$	$w = \frac{3630\,s^3}{s+6D}$
Oak, English ..	$w = 3960\,bd$	$w = 990\,bd$	$w = \frac{3960\,bd^2}{d+6D}$	$w = 3960\,s^2$	$w = 990\,s^2$	$w = \frac{3960\,s^3}{s+6D}$
Pine, American	$w = 3900\,bd$	$w = 975\,bd$	$w = \frac{3900\,bd^2}{d+6D}$	$w = 3900\,s^2$	$w = 975\,s^2$	$w = \frac{3900\,s^3}{s+6D}$

We have hitherto taken no notice of the column when its form is cylindrical; but from the relation of

its strength to that of its circumscribed prism, the formulæ may be easily adapted to this form also. Now, it has been shewn in the solution of Prob. III. page 10, that the strength of a cylindric beam, is to the strength of its circumscribing square prism as 4·71 : 8 very nearly; but it is further shewn by writers on mechanics, that in the extreme limit of strength the ratio is as 5·5 to 8 nearly; the mean is therefore 5·1 to 8: and by reducing the equations for the square column in the same ratio, the results will very nearly represent the strength of cylindrical columns under the same, or similar circumstances. The reduced results are as follows.

For short Cylindrical Columns.

	Cast Iron.	Wrought Iron.
1	$w = 9754\, d^2$	$w = 11348\, d^2$
2	$w = 2439\, d^2$	$w = 2837\, d^2$
3	$w = \frac{9754\, d^3}{d + 6\text{D}}$	$w = \frac{11348\, d^3}{d + 6\text{D}}$

The method of reducing these equations is the same as we have already illustrated, and when modified for the several kinds of wood, they are as exhibited in the following table.

Table of Formulæ for Cylindrical Columns.

Name of the Wood.	Case 1.	Case 2.	Case 3.
Ash..........	$w = 2257d^2$	$w = 564d^2$	$w = \frac{2257\,d^3}{d + 6D}$
Beech	$w = 1504d^2$	$w = 376d^2$	$w = \frac{1504\,d^3}{d + 6D}$
Elm	$w = 2065d^2$	$w = 516d^2$	$w = \frac{2065\,d^3}{d + 6D}$
Fir, red or yellow	$w = 2735d^2$	$w = 684d^2$	$w = \frac{2735\,d^3}{d + 6D}$
Fir, white	$w = 2314d^2$	$w = 579d^2$	$w = \frac{2314\,d^3}{d + 6D}$
Oak, English ..	$w = 2524d^2$	$w = 631d^2$	$w = \frac{2524\,d^3}{d + 6D}$
Pine, American	$w = 2486d^2$	$w = 622d^2$	$w = \frac{2486\,d^3}{d + 6D}$

The following example will be sufficient to shew how these formulæ are to be applied to the calculation of strength.

Example 4. A cylindrical column of beech whose length is inconsiderable, has its diameter 8 inches; what weight will it bear, the direction of the straining force being perpendicular to the plane of fracture, and one inch distant from the centre of the column?

The formula for beech in this case, is

$$w = \frac{1504\,d^3}{d+6D}.$$

Here $d=8$ inches and $D=1$ inch; therefore, by substitution we get

$$w = \frac{1504 \times 8^3}{8+6} = 55003 \text{ lbs.}$$ the weight required nearly.

We have now to consider the resistance of columns, when they are of such a length, that the deflexion produced by the strain may be sufficient to increase the distance of the direction of the straining force from the axis of the column; for in that case, it is evident that the strain will be increased. Now, the writers on the resistance of solids have shewn, that the deflexion in the middle of a column, when strained by a compressing force, is a maximum when the neutral axis coincides with the axis of the column (*see* TREDGOLD on Cast Iron, Art. 240); and in that case the calculated deflexion is

$$\delta = \frac{l^2}{4816\,d}.$$

But the distance of the straining force from the axis of the column is increased by that quantity, that is

$$D + \frac{l^2}{4816\,d}.$$

Let this be substituted for D in equation (*e*), or rather, in the third case for short rectangular columns, and it becomes, in the case of strength

$$w = \frac{15300\,b\,d^2}{d + 6\,D + \frac{6\,l^2}{4816\,d}}.$$

But in order that the length may be in feet, we have

$$w = \frac{15300\,b\,d^2}{d + 6\,D + \frac{54\,l^2}{301\,d}}.$$

Which, after further reduction, becomes

$$w = \frac{4605300\,b\,d^3}{301\,d^2 + 1806\,d\,D + 54\,l^2} \quad \ldots\ldots (f)$$

Therefore, from this equation, in the case of columns of considerable length, we have,

1. *When the direction of the straining force coincides with the axis of the column.*

In this case $D = 0$; hence, by substitution and reduction, equation (f) becomes

$$w = \frac{15300\, b\, d^3}{d^2 + \cdot 179\, l^2}.$$

2. *When the direction of the straining force coincides with the surface of the column.*

Here we have $D = \frac{1}{2} d$; therefore, by substitution, equation (f) becomes

$$w = \frac{3825\, b\, d^3}{d^2 + \cdot 045\, l^2}.$$

3. *When the direction of the straining force is perpendicular to the plane of fracture, but not coincident with the axis or surface of the column.*

In this case equation (f) becomes

$$w = \frac{15330\, b\, d^3}{d^2 + 6\, d\, D + \cdot 179\, l^2}.$$

These equations are applicable to columns of a rectangular form, and when adapted to the square prism and the cylinder, they are exhibited in the following table.

	Square Columns.	*Cylindrical Columns.*
1	$w = \frac{15300\, s^4}{s^2 + \cdot 179\, l^2}$	$w = \frac{9754\, d^4}{d^2 + \cdot 179\, l^2}$
2	$w = \frac{3825\, s^4}{s^2 + \cdot 045\, l^2}$	$w = \frac{2439\, d^4}{d^2 + \cdot 045\, l^2}$
3	$w = \frac{15300\, s^4}{s^2 + 6\, d\, D + \cdot 179\, l^2}$	$w = \frac{9754\, d^4}{d^2 + 6\, d\, D + \cdot 179\, l^2}$

It must be observed, however, that the expressions for the cylindrical columns are only approximative, on account of having employed the mean between the two limits of strength, instead of the true number that would result from a direct solution of the particular case; but they will, notwithstanding, be found sufficiently near the truth for practical purposes.

When modified for wrought iron, they are as follows:

Table of Formulæ for Wrought Iron Columns of the Rectangular, Square, and Cylindrical Forms.

	Rectangular Columns.	*Square Columns.*	*Cylindrical Columns.*
1	$w = \frac{17800\, b\, d^3}{d^2 + \cdot 154\, l^2}$	$w = \frac{17800\, s^4}{s^2 + \cdot 154\, l^2}$	$w = \frac{11348\, d^4}{d^2 + \cdot 154\, l^4}$
2	$w = \frac{4450\, b\, d^3}{d^2 + \cdot 038\, l^2}$	$w = \frac{4450\, s^4}{s^2 + \cdot 338\, l^2}$	$w = \frac{2837\, d^4}{d^2 + \cdot 038\, l^2}$
3	$w = \frac{17800\, b\, d^3}{d^2 + 6 d \mathrm{D} + \cdot 154\, l^2}$	$w = \frac{17800\, s^4}{d^2 + 6 d \mathrm{D} + \cdot 154\, l^2}$	$w = \frac{11348\, d^4}{d^2 + 6 d \mathrm{D} + \cdot 154\, l^2}$

The adaptation of these formulæ to the several kinds of wood, the cohesive force of which is known, is left for exercise to the reader; we therefore proceed to apply those for cast and wrought iron to a few practical examples.

Example 5. A rectangular column of cast iron, 2 inches deep, 5 inches broad, and 24 feet long, is strained to the extent of its elastic force, by a load

acting in the direction of the axis; by what load is it compressed?

This is the first case of rectangular columns, the formula for which, is

$$w=\frac{15300\,b\,d^3}{d^2+\cdot 179\,l^2}.$$

Here $d=2$ inches, $b=5$ inches, and $l=24$ feet; let these numbers be substituted for b, d and l in the equation, and it becomes

$$w=\frac{15300\times 5\times 2^3}{2^2+\cdot 179\times 24^2};$$

that is, $w=\frac{612000}{107\cdot 104}=5714$ lbs. the load required.

Example 6. What is the side of a square beam of the same length, and placed under similar circumstances, to bear the same load?

The formula for this case of a square column, is

$$w=\frac{15300\,s^4}{s^2+\cdot 179\,l^2}.$$

But $w=5714$ lbs. and $l=24$ feet; hence, by substitution, we have

$$5714=\frac{15000\,s^4}{s^2+103\cdot 104};$$ therefore, by multiplication and transposition, we obtain

$$15300s^4-5714s^2=589136\cdot 256,$$

and this by division becomes

$$s^4-\cdot 3734s^2=38\cdot 5056,$$

and by completing the square we get

$$s^4-\cdot 3734s^2+\cdot 03486=38\cdot 54046,$$

and, by evolution, it becomes

$$s^2 - \cdot1867 = 6\cdot208,$$ which, by transposition, is

$$s^2 = 6\cdot3947\ ;$$

therefore, by evolution, we have 3947

$$s = \sqrt{6\cdot\cancel{208}} = 2\cdot\cancel{49}$$ inches, the side of the square required. 52

The above operation is tedious, but it cannot be avoided in cases where the length and the load are given to determine the side; a similar process discovers the diameter of a cylindrical column; but in the case of a rectangle, if the breadth, together with the length and the load, be given to determine the depth, the process is still more tedious and difficult, as, in that case, the reduction of a cubic equation is necessary. It is not, however, in the power of science to simplify the subject; and for those who are unacquainted with algebra, there is no alternative but having recourse to tables constructed for the purpose; or, otherwise, where tables are not to be found, their works, however extensive and important, must in a great measure be constructed at random.

Example 7. Required the weight that a cylindrical column of cast iron, 6 inches in diameter and 14 feet long, can support with safety, the direction of the straining force coinciding with the surface?

The formula for this case of a cylindrical column, is

$$w = \frac{2439\,d^4}{d^2 + \cdot 045\,l^2}.$$

Here we have $d=6$ inches, and $l=14$ feet; let these values be substituted for d and l in the equation, and it becomes

$$w = \frac{2439 \times 6^4}{6^2 + \cdot 045 \times 14^2};$$

that is, $w = \frac{3160944}{44 \cdot 82} = 70525$ lbs. the weight required.

Mr. Tredgold's constant for this case is 2390, which, of course, would give a much less load: the difference of the results arises from the circumstance of his having taken the mean of the limiting ratios a little in excess; but this, after all, is perhaps the safest way in practice.

Example 8. The length of a rectangular column of wrought iron is 16 feet, its breadth 4, and depth $2\frac{1}{2}$ inches; what load will it sustain, supposing the direction of the straining force to be half an inch distant from the axis?

The formula for this case of wrought iron, is

$$w = \frac{17800\,b\,d^3}{d^2 + 6\,d\,D + \cdot 154\,l^2}.$$

But $b=4$ inches, $d=2\frac{1}{2}$ inches, $D=\frac{1}{2}$ an inch, and $l=16$ feet; let these numbers be substituted for b, d, D and l, and our equation becomes

$$w = \frac{17800 \times 4 \times 2 \cdot 5^3}{2 \cdot 5^2 + 6 \times 2 \cdot 5 \times \cdot 5 + \cdot 154 \times 16^2};$$

that is, $w = \frac{1112500}{59 \cdot 574} = 18674$ lbs. the load required.

Another example or two will suffice for this part of the subject.

Example 9. What force will a circular rib of cast iron resist, acting in the direction of its chord, the length of the chord being 12 feet, the breadth of the rib 9 inches, its depth or thickness 2 inches, and the greatest distance of the chord from the axis of the rib $5\frac{3}{4}$ inches?

This agrees with the third case of rectangular beams, where the direction of the straining force is not coincident with the axis or surface of the beam.

The formula for this case, is

$$w=\frac{15300\,b\,d^3}{d^2+6d\text{D}+\cdot179\,l^2}.$$

Here $b=9$ inches, $d=2$ inches, $\text{D}=5\cdot75$ inches, and $l=12$ feet; let these values of b, d, D and l, be substituted instead of them in the preceding equation, and it becomes

$$w=\frac{15300\times9\times2^3}{2^2+6\times2\times5\cdot75+\cdot179\times12^2};$$

that is, $w=\frac{1101600}{98\cdot776}=11152$ lbs, the force required.

Example 10. A cylindrical rod of wrought iron, 4 feet long, is exposed to a compression or tension of 5760 lbs. in the direction of its axis; what is its diameter?

This is the first case of cylindrical beams, and the formula for wrought iron, is

$$w=\frac{11348\,d^4}{d^2+\cdot154\,l^2}.$$

Here we have $w = 5760$ lbs. and $l = 4$ feet, by the question; let these numbers be substituted in the equation, and it becomes

$$5760 = \frac{11348 d^4}{d^2 + 2{\cdot}864};$$ and this, by exterminating the fraction, becomes

$$5760 d^2 + 16496{\cdot}64 = 11348 d^4,$$

or by transposition we have

$$11348 d^4 - 5760 d^2 = 16496{\cdot}64;$$

and again by division it is

$$d^4 - {\cdot}5075\, d^2 = 1{\cdot}4537;$$ therefore, by completing the square we obtain,

$$d^4 - {\cdot}5075\, d^2 + \left(\frac{{\cdot}5075}{2}\right)^2 = 1{\cdot}4537 + \left(\frac{{\cdot}5075}{2}\right)^2,$$

and by extracting the square root we get

$$d^2 - \frac{{\cdot}5075}{2} = \sqrt{1{\cdot}4537 + \left(\frac{{\cdot}5075}{2}\right)^2} = 1{\cdot}2295;$$

and again by transposition we obtain,

$$d^2 = 1{\cdot}4832;$$

therefore, $d = \sqrt{1{\cdot}4833} = 1{\cdot}21$ inches, the diameter sought.

Such is the method of calculating the piston rods for double-acting steam engines; they are alternately exposed to compression and tension, and the force which is to be resisted is the pressure of the steam on the piston.

We have stated elsewhere, that when the beam is exposed to a compressive force, inasmuch as the flexure increases the direction of the force from the

axis of the beam, so much is the strain increased; on the contrary, however, when the beam is exposed to an extending force, we find that the strain, instead of being increased by flexure, is diminished by it, inasmuch as it decreases the distance of the extending force from the axis of the beam. This is an important circumstance in the application of iron supports: it shews, that in all cases where it is possible to employ a tensile strain, the force of the metal to resist fracture is so much the greater, and, in consequence, the fitter for its intended purpose. We have already shewn that the flexure in the case of compression is expressed by the equation

$$\delta = \frac{l^2}{4816\,d};$$

but since bodies resist compression and extension with equal forces, it is obvious that the flexure in the case of extension must be the same; therefore, if l the length of the beam be taken in feet, we have

$$\delta = \frac{9\,l^2}{301\,d}.$$

But D is the distance of the direction of the straining force from the axis of the beam, and

$$D - \frac{9\,l^2}{301\,d},$$

becomes that distance when modified for the effect of flexure. Let this expression be substituted for D in the third case for short rectangular beams, and we get

$$w = \frac{15300\,b\,d^2}{d + 6\,\mathrm{D} - \frac{54\,l^2}{301\,d}};$$

which, by reducing the fraction in the last member of the denominator, becomes

$$w = \frac{15300\,b\,d^3}{d^2 + 6\,\mathrm{D} - \cdot 179\,l^2} \ldots\ldots (g)$$

therefore, to allow for the effect of flexure in beams of considerable length, when exposed to an extending force, we have

1. *When the direction of the straining force coincides with the axis.*

Here the distance between the direction of the straining force is nothing, consequently the representative of that distance in the denominator of the fraction which expresses the strength is nothing also; therefore we obtain

$$w = 15300\,bd.$$

We may here observe, that the result just obtained is precisely the same as that for the first case of short rectangular beams; from which we infer, that when a body is compressed or extended in the direction of its axis, the strength is neither increased nor diminished by means of its length, setting aside the effect that may be produced by the weight of the beam itself.

2. *When the direction of the straining force coincides with the surface of the column.*

In this case, the direction of the straining force is distant from the axis of the beam by half its depth;

that is, $D = \frac{1}{2} d$; let this value of D be substituted in equation (*g*), and we get

$$w = \frac{3825\, b\, d^3}{d^2 - \cdot 045\, l^2}.$$

3. *When the direction of the straining force is perpendicular to the plane of fracture, but not coincident with the axis or surface of the beam.*

The equation for this case is the same as equation (*g*), that is,

$$w = \frac{15300\, b\, d^3}{d^2 + 6\, D - \cdot 179\, l^2}.$$

The adaptation of these equations to square and cylindrical beams, and also to the same forms of wrought iron, is the very same as has been done already, observing only, that in the case of a tensile flexure, the last member of the denominator of the fraction which expresses the strength must be subtracted.

We may here remark, that what we have hitherto done applies only to particular cases, where the direction of the straining force is perpendicular to the plane of fracture; but there may be other cases where this will not happen, and consequently the foregoing equations will not apply; the general solution, however, is indicated in equation (*e*), which being modified for flexure, as respects the compressing and extending strain, becomes

$$w = \frac{15300\, b\, d^3 \operatorname{cosec} \varphi}{d^2 + 6\, d\, D \pm \cdot 179\, l^2 + 36\, l \cot \varphi} \ldots \ldots (h)$$

For the square beam, it is

$$w = \frac{15300\, s^4 \operatorname{cosec} \varphi}{s^2 + 6\, s\, D \pm \cdot 179\, l^2 + 36\, l \cot \varphi},$$

and for the cylindrical beam it is

$$w = \frac{9754\, d^4 \operatorname{cosec} \phi}{d^2 + 6\, d\, D \pm \cdot 179\, l^2 + 36\, l \cot \varphi}.$$

The adaptation of these general equations to wrought iron is left for the reader's practice; and by carefully attending to what has been done, it is presumed he will find no difficulty in doing so: we shall therefore conclude our inquiries respecting the *resistance to compression and tension* with the solution of the following general example.

Example 11. A uniform rectangular beam of cast iron, 4 inches broad, 3 inches deep, and 12 feet long, is strained to the extent of its elastic power by a load which compresses it in a direction inclined to the plane of fracture at an angle of 89°; what load does it sustain?

In this example the distance of the direction of the straining force from the axis of the beam is, by plane trigonometry,

$$D = 6 \cot \varphi.$$

Therefore, by substituting this expression for D in equation (h), it becomes for compression

$$w = \frac{15300\, b d^3 \operatorname{cosec} \varphi}{d^2 + 36(d + l) \cot \varphi + \cdot 179 l^2}.$$

Now, $b = 4$ inches, $d = 3$ inches, and $l = 12$ feet; let these numbers be substituted for b, d, and l, and the equation becomes

$$w = \frac{1652400 \operatorname{cosec} \varphi}{24 \cdot 776 + 540 \cot \varphi};$$

but $\varphi = 89^\circ$; consequently $\operatorname{cosec} \varphi = 1$ nearly, and $\cot \varphi = \cdot 01745$. Hence we get

$$w = \frac{1652400}{35 \cdot 2} = 46943 \text{ lbs. the load}$$

required.

It is much to be regretted that the subject involves such high equations in the determination of the dimensions to resist a given strain under certain conditions; because, from the small share of mathematical knowledge possessed by mechanics in general, we fear that a direct calculation in this respect will seldom be resorted to; and an approximate estimate made indirectly, may frequently be attended with very serious consequences: we would therefore earnestly advise every person connected with mechanical pursuits, to pay attention to the study of algebra, in order that he may be prepared to resolve all such equations whenever they occur; for it is impossible to adapt the deductions of science to the capacities of those who have occasion for them. It is however to be expected, that the formation of mechanics' institutions and mechanical schools, will have an important influence in extending mathematical knowledge among the members of the profession.

We now proceed to consider the resistance of bodies when exposed to impulsive forces.

RESISTANCE TO IMPULSION.

RESISTANCE TO IMPULSION.

THE forms of beams which are most likely to be exposed to impulsive forces in cases of practice are the *rectangular*, the *grooved*, and the *open*; therefore, in considering the nature of *resistance to impulsion*, we shall confine our inquiries to these three forms only.

Now, we have shewn in the solution of the *first*, *fifth*, and *sixth* problems, that the expressions for the resistance of those forms in the case of pressure are respectively as below, viz.

For the *rectangular* beam, $l\,w = 850\,b\,d^2$,

For the *grooved* beam, - $l\,w = 850\,b\,d^2(1-q\,p^3)$,

For the *open* beam, - - $l\,w = 850\,b\,d^2(1-p^3)$.

And in the solution of the tenth problem it is shewn that the deflexion for those forms is,

$$\delta = \frac{\cdot 02\,l^2}{d}, \text{ or rather } \delta = \frac{144\,l^2}{7225\,d}.$$

Hence, by taking the deflexion in feet, and introducing the number which expresses the power of gravity, viz. $32\frac{1}{6}$; we have, from the laws of the collision of bodies,

$$m\,v = \sqrt{\frac{772\,b\,d\,l\,m}{17}}.$$

And, by squaring both sides of this equation, it becomes, after proper reduction,

$$17\,m v^2 = 772\,b\,d\,l;$$

where v is the velocity of the moving body in feet per second, m its mass or weight in lbs., and b, d, and l, the dimensions of the beam as formerly employed: hence, when the moving body is urged by the power of gravity, we have,

1. *When the beam is uniform in breadth and depth,*

$$17\,m v^2 = 772\,b\,d\,l.$$

2. *When the beam is uniform in depth, and its transverse section in form of the letter* I.

$$17\,m v^2 = 772\,b\,d\,l(1 - q p^3).$$

3. *When the beam is uniform in breadth and depth, but has a part of the middle left out,*

$$17\,m v^2 = 772\,b\,d\,l\,(1 - p^3).$$

These three cases express the conditions of the beams when the depth is the same throughout the length; but, as we have remarked elsewhere, those forms are, for the sake of lightness and economy, frequently cast with the outline of the depth elliptical, and when this happens to be the case, the equations which express the conditions of resistance, are as under.

4. *When the breadth is uniform, and the outline of the depth elliptical,*

$$17\,m v^2 = 992\,b\,d\,l.$$

5. *When the outline of the depth is elliptical, and the transverse section in form of the letter* I,

$$17\, m v^2 = 992\, b\, d\, l\, (1 - q p^3).$$

6. *When the outline of the depth is elliptical, and part of the middle left out.*

$$17\, m v^2 = 992\, b\, d\, l\, (1 - p^3).$$

The last three of these equations are deduced from the first three in the following manner. It has been shewn in Problem X. that the deflexion for a uniform beam, when the load is applied at the middle of the length, is

$$\delta = \frac{\cdot 144\, l^2}{7225\, d};$$

and at page 46 it is shewn, that the deflexion for a beam having the outline of the depth an ellipsis, is

$$\delta = \frac{\cdot 0257\, l^2}{d};$$

or, more accurately,

$$\delta = \frac{370 \cdot 224\, l^2}{14450\, d};$$

therefore we have

$$\frac{144}{7225} : \frac{370 \cdot 224}{14450} :: 772 : 992 \text{ very nearly.}$$

Hence, 992 being substituted for 772 in each of the first three equations, gives the last three accordingly.

We may further observe, that in each of the preceding six cases, the beam is supposed to be supported at the ends, and the load to produce its effect at the middle point; but in the case of the three latter, this supposition is not necessary; for, as we

shall shew further on, the beams are alike capable at every point throughout the length to resist the impulsion, and hence they are designated *beams of equal strength*.

We shall, in the next place, give a few examples to shew the method of reducing the formulæ.

Example 1. A uniform rectangular beam of cast iron, 1·53 inches broad, 6 inches deep, and 26 feet long, is exposed to a gravitating force of 8 feet per second; what is the weight of the gravitating body, the impulse being just within the elastic power of cast iron?

The formula belongs to Case 1, viz.

$$17\, m\, v^2 = 772\, b\, d\, l.$$

Now $b = 1·53$ inches, $d = 6$ inches, $l = 26$ feet, and $v = 8$ feet: let these numbers be substituted in the equation for $b, d, l,$ and v, and it becomes

$$17 \times 8^2 \times m = 772 \times 1·53 \times 6 \times 26;$$

that is, $m = \frac{184260·96}{1088} = 169·35$ lbs. very nearly.

But by the first case of rectangular beams, when exposed to pressure, a beam of the same section, and 26 feet long, will bear, without permanent alteration, a load of not less than 1801 lbs. including its own weight, or 1418 lbs. excluding its own weight; hence it appears, that a load of 169 lbs. falling from a height of about one foot, will produce the same effect on a beam by impulsion, that a load of 1418 lbs.

will produce by pressure, setting aside the consideration of the effect produced by the weight of the beam in both cases, but which would indeed have a very important influence in the comparison.

Example 2. A cast iron beam of uniform depth, having its transverse section in form of the letter I, is supported at the ends, and exposed to a blow from a weight of 100 lbs. falling freely through a height of 4 feet; what must be the depth of the beam to sustain the shock, supposing its length to be 30 feet, its greatest breadth 4 inches, the breadth in the middle $1\frac{1}{2}$ inches, and the depth in the middle .7 times the whole depth?

If h be the height in feet through which a body falls by the force of gravity, then, the writers on the laws of falling bodies have shewn that,

$$v^2 = 64\tfrac{1}{3} h.$$

Consequently, when the height of the fall is given instead of the velocity, we have only to employ $64\frac{1}{3}$ times that height in the calculation instead of v^2, taking care, however, that the fall is reduced to feet, or otherwise, $64\frac{1}{3}$ must be reduced to inches, in order to render the terms homogeneous.

In the solution of the fifth problem it is shewn that $p = \frac{D}{d}$, and $q = \frac{b - B}{b}$; where B is the breadth of the middle or grooved part of the beam, and D its

depth; consequently, in the present example we have $p = \frac{D}{d} = \cdot 7$, and $q = \frac{4 - 1\frac{1}{2}}{4} = \cdot 625$. Now, the formula for this example belongs to Case 2, and is

$$17\, m\, v^2 = 772\, b\, d\, l(1 - q\, p^3).$$

If the above values of p and q be substituted in this equation, it becomes

$$17\, m\, v^2 = 606\, b\, d\, l.$$

Now, $b = 4$ inches, $l = 30$ feet, $m = 100$ lbs. and $v^2 = 64\frac{1}{3} \times 4$; let these numbers be substituted in the equation, and we obtain

$$606 \times 4 \times 30 \times d = 17 \times 100 \times 64\tfrac{1}{3} \times 4;$$

$$\text{that is, } d = \frac{32810}{5454} = 6\cdot 01 \text{ inches nearly,}$$

the depth required. Therefore, the depth of the middle part or groove is $6\cdot 01 \times \cdot 7 = 4\cdot 207$ inches.

The fractions ·7 and ·625 are those recommended by Mr. Tredgold as answering very well for practical purposes; but it may often happen, that the value both of p and q may vary from the above numbers; and as the substitution of those quantities in the original equation is tedious, on account of the high exponent of p, we shall, before we proceed further, give a small tablet of constants to be used in the equation for all values of p and q, from ·1 to ·9, both inclusive, as follows.

Table of Constants corresponding to the several Values of p and q.

Values of q.	Values of the Fraction p.								
	·1	·2	·3	·4	·5	·6	·7	·8	·9
·1	772	771	770	767	762	755	745	732	716
·2	772	771	768	762	753	738	719	693	659
·3	772	770	766	757	743	722	692	653	603
·4	772	769	764	752	733	705	666	614	547
·5	772	769	761	747	724	688	639	574	490
·6	771	768	759	742	714	672	613	535	434
·7	771	767	757	737	704	655	586	495	378
·8	771	767	755	732	695	638	560	456	322
·9	771	766	753	727	685	622	534	416	265
1·0	771	766	751	723	675	605	507	377	209

The use of this table in the practice of calculation may be exemplified as follows.

Example 3. A grooved beam of cast iron, of uniform depth throughout, is 5 inches broad in the projections, $2\frac{1}{2}$ inches broad in the groove, 24 inches the whole depth, 19·2 inches deep in the groove, and 36 feet long; what weight will it sustain, falling on its middle point with a velocity of 10 feet per second?

Here $p = \frac{19{\cdot}2}{24} = {\cdot}8$, and $q = \frac{5 - 2{\cdot}5}{5} = {\cdot}5$; and the formula is that belonging to Case 2, viz.

$$17\, m v^2 = 772\, b\, d\, l(1 - q p^3).$$

Therefore, look in the table under $\cdot8$, the value of p, and opposite $\cdot5$, the value of q; and in the angle of meeting we find 574 for the constant to be employed in the preceding equation instead of $772\,(1-q\,p^3)$; hence we get

$$17\,m\,v^2 = 574\,b\,d\,l.$$

Now, $b = 5$ inches, $d = 24$ inches, $l = 36$ feet, and $v = 10$; let these numbers be substituted in the equation reduced for p and q, and it becomes

$$17 \times 10^2 \times m = 574 \times 5 \times 24 \times 36;$$

$$\text{that is, } m = \frac{2479680}{1700} = 1458{\cdot}6 \text{ lbs. nearly,}$$

the weight required. And so on for any other value of p and q within the limits of the Table.

Example 4. A load of 120 lbs. falling freely by the force of gravity, strikes a cast iron beam with a velocity of 20 feet per second; what must be the depth of the beam to sustain the shock, its breadth being 4 inches, length 30 feet, and the depth of the open part $\cdot6$ times the whole depth?

The formula is that belonging to Case 3, viz.

$$17\,m\,v^2 = 772\,b\,d\,l\,(1-p^3).$$

Now, in this case $p = \cdot6$ given, $q = 1$, from the nature of the question; therefore, in the Table under $\cdot6$, the value of p, and opposite 1, the value of q, we have 605, the constant to be employed instead of $772\,(1-p^3)$; hence the equation, reduced for p, becomes

$$17\,m\,v^2 = 605\,b\,d\,l.$$

But $b = 4$ inches, $l = 30$ feet, $m = 120$ lbs. and $v = 20$ feet; let these numbers be substituted in the reduced equation, and we have

$$605 \times 4 \times 30 \times d = 17 \times 120 \times 20^2;$$

$$\text{that is, } d = \frac{6800}{605} = 11{\cdot}23 \text{ inches nearly,}$$

the depth required; and $11{\cdot}23 \times {\cdot}6 = 6{\cdot}738$ inches, the depth of the open part.

The following tablet will assist the reader in applying the formula for this form of beam for the several values of the letter p from ·1 to ·9, both inclusive.

$$p = {\cdot}1 \ldots.17\,m\,v^2 = 771\,b\,d\,l,$$
$$p = {\cdot}2 \ldots.17\,m\,v^2 = 766\,b\,d\,l,$$
$$p = {\cdot}3 \ldots.17\,m\,v^2 = 751\,b\,d\,l,$$
$$p = {\cdot}4 \ldots.17\,m\,v^2 = 723\,b\,d\,l,$$
$$p = {\cdot}5 \ldots.17\,m\,v^2 = 675\,b\,d\,l,$$
$$p = {\cdot}6 \ldots.17\,m\,v^2 = 605\,b\,d\,l,$$
$$p = {\cdot}7 \ldots.17\,m\,v^2 = 507\,b\,d\,l,$$
$$p = {\cdot}8 \ldots.17\,m\,v^2 = 377\,b\,d\,l,$$
$$p = {\cdot}9 \ldots.17\,m\,v^2 = 209\,b\,d\,l;$$

where it will be seen that the constants are respectively those in the last line of the preceding table, in which case $q = 1$, as it must be in an open beam, where the breadth of the middle part is zero.

Example 5. A load of 2240 lbs. is allowed to fall

from a height of 9 inches, on the middle point of an open beam of cast iron, whose depth is 28 inches, and length 34 feet; what must be the breadth of the beam to resist the shock, the depth being uniform, and the depth of the open part ·5 times the whole depth?

Here, $p = \cdot 5$, consequently the formula for this example, taken from the preceding tablet, is

$$17\,m\,v^2 = 675\,b\,d\,l.$$

Now, we have already shewn that when the height of the fall is given in feet,

$$v^2 = 64\tfrac{1}{3}\,h\,;$$

but in this instance $h = \cdot 75$ feet; hence we get $v^2 = 48\frac{1}{4}$; and, by the question, $d = 28$ inches, $l = 34$ feet, and $m = 2240$ lbs. hence by substitution we obtain

$$675 \times 28 \times 34 \times b = 17 \times 2240 \times 48\tfrac{1}{4}\,;$$

that is, $b = \dfrac{1837360}{642600} = 2 \cdot 86$ inches very nearly, for the breadth required; $28 \times \cdot 5 = 14$ inches, for the depth of the open part.

We have next to consider those cases where the outline of the depth is elliptical, as follows:

Example 6. A cast iron beam, 30 feet long and 2 inches broad, has the outline of its depth an ellipsis; what must be its greatest depth to resist a load of 3360 lbs. falling from a height of 3 inches?

The formula for this example is that belonging to Case 4, viz.

$$17\, m\, v^2 = 992\, b\, d\, l.$$

But $b = 2$ inches, $l = 30$ feet, $m = 3360$ lbs., and $v^2 = 64\frac{1}{3} \times \frac{1}{4}$; let these numbers be substituted in the equation for b, l, m, and v, and it becomes

$$992 \times 2 \times 30 \times d = 17 \times 3360 \times 64\tfrac{1}{3} \times \tfrac{1}{4};$$

$$\text{that is, } d = \frac{918680}{59520} = 15{\cdot}43 \text{ inches}$$

nearly, the depth required.

Example 7. A cast iron beam, 25 feet long, has the outline of the depth an ellipsis, its greatest breadth 6 inches, breadth of the middle part $2\frac{1}{2}$ inches, the greatest depth 20 inches, and the depth of the middle part at the place of greatest section 15 inches; from what height must a load of 224 lbs. descend, so that the force of impact may be just within the elastic power of the beam?

The formula for this example belongs to Case 5, viz.

$$17\, m\, v^2 = 992\, b\, d\, l\,(1 - q\, p^3).$$

Now, $p = \frac{15}{20} = {\cdot}75$, and $q = \frac{6 - 2{\cdot}5}{6} = {\cdot}58\frac{1}{3}$; therefore, by modifying the equation for these values of p and q, it becomes

$$17\, m\, v^2 = 742\, b\, d\, l.$$

But $b = 6$ inches, $d = 20$ inches, $l = 25$, and $m = 224$ lbs. let these numbers be substituted in the reduced equation, and we get

$$17 \times 224 \times v^2 = 742 \times 6 \times 20 \times 25;$$

$$\text{that is, } v^2 = \frac{2226000}{3708} = 600{\cdot}3209,$$

$$\text{or, } v = \sqrt{600{\cdot}3209} = 24{\cdot}5 \text{ feet per}$$

second nearly, for the velocity of impact, which corresponds to a fall of 9·33 feet nearly.

The following Table shews the constants to be used for all values of p and q, from ·1 to 1, both inclusive.

Table of Constants corresponding to the several values of p and q.

Value of q.	Values of the Fraction p.								
	·1	·2	·3	·4	·5	·6	·7	·8	·9
·1	992	991	989	985	979	970	958	941	919
·2	992	990	987	979	967	949	924	890	847
·3	992	989	984	973	955	928	890	839	775
·4	992	989	981	967	942	906	856	789	703
·5	991	988	978	960	930	885	822	738	630
·6	991	987	976	954	917	863	788	687	588
·7	991	986	973	947	905	842	754	636	486
·8	991	985	970	941	893	820	720	586	413
·9	991	985	968	935	880	799	686	535	341
1·0	991	984	965	928	868	778	652	484	269

The use of the Table for practical purposes, will become manifest from the following example.

Example 8. A cast iron beam, 32 feet long and

4 inches broad, has the outline of its depth an ellipsis, and is subjected to an impulsive force of 8 feet per second; what is the weight of the falling body, supposing the greatest depth of the beam to be 36 inches, the depth of the middle part 28·8 inches, and breadth 2 inches?

Here, $p=\frac{28\cdot8}{36}=\cdot8$ inches, and $q=\frac{4-2}{4}=\cdot5$ inches; therefore, in the table under ·8, the value of p, and opposite ·5, the value of q, we find 738 for the constant to be employed in the equation, instead of 992 $(1-qp^3)$, to which it is equivalent.

The formula, therefore, is

$$17\,m\,v^2=738\,b\,d\,l.$$

Now, $b=4$ inches, $d=36$ inches, $l=32$ feet, and $v=8$ feet; let these numbers be substituted in the equation for b, d, l and v, and it becomes

$$17\times8^2\times m=738\times4\times36\times32;$$

$$\text{that is, } m=\frac{53136}{17}=3125\cdot6 \text{ lbs. very nearly,}$$

for the weight of the falling body required.

Example 9. An open beam of cast iron, 28 feet long and 5 inches broad, has the outline of the depth elliptical, and is exposed to the force of 500 lbs falling from a height of 9 feet; what must be the greatest depth of the beam to sustain the blow, the depth of the open part being ·7 of the whole depth?

The formula for this example is that belonging to Case 6, viz.

$$17\,m\,v^2 = 992\,b\,d\,l\,(1-p^3).$$

But $p = {\cdot}7$; therefore by substitution our equation becomes

$$17\,m\,v^2 = 652\,b\,d\,l.$$

Now, $b=5$ inches, $l=28$ feet, $m=500$ lbs. and $v^2 = 64\frac{1}{3} \times 9$; let these numbers be substituted in the equation instead of b, l, m and v, and we have

$$652 \times 5 \times 28 \times d = 17 \times 500 \times 64\tfrac{1}{3} \times 9;$$

$$\text{that is, } d = \frac{4921500}{91280} = 53{\cdot}9 \text{ inches nearly,}$$

for the depth required; the depth of the open part is $53{\cdot}9 \times {\cdot}7 = 37{\cdot}73$ inches, consequently the deepness of metal is only $53{\cdot}9 - 37{\cdot}73 = 16{\cdot}17$ inches. Now, a beam of this form, and of the dimensions thus determined, will sustain a load of 285554 lbs. when exposed to pressure only; from which appears the great disadvantage of exposing beams of such a brittle nature as cast iron is, to very high impulsive forces.

The following tablet for the several values of p, will be useful in the calculation of beams of this form.

$$p = {\cdot}1 \ldots . 17\,m\,v^2 = 991\,b\,d\,l,$$
$$p = {\cdot}2 \ldots . 17\,m\,v^2 = 984\,b\,d\,l,$$
$$p = {\cdot}3 \ldots . 17\,m\,v^2 = 965\,b\,d\,l,$$
$$p = {\cdot}4 \ldots . 17\,m\,v^2 = 928\,b\,d\,l,$$

$$p = \cdot 5 \ldots\ldots 17\, m v^2 = 868\, b\, d\, l,$$
$$p = \cdot 6 \ldots\ldots 17\, m v^2 = 778\, b\, d\, l,$$
$$p = \cdot 7 \ldots\ldots 17\, m v^2 = 652\, b\, d\, l,$$
$$p = \cdot 8 \ldots\ldots 17\, m v^2 = 484\, b\, d\, l,$$
$$p = \cdot 9 \ldots\ldots 17\, m v^2 = 269\, b\, d\, l.$$

We shall, in the next place, endeavour to shew the advantage that arises from moulding the beams into the elliptical form, after the manner implied in the *fourth*, *fifth*, and *sixth* cases preceding.

It is stated in the Note to Case 2 of the first problem on the *transverse strain*, that it is an important circumstance in the construction of cast iron beams, the strain being proportional to the rectangle of the segments into which the length between the supports is divided at the point where the load is applied: from it we learn, that the depth of the cross section may be diminished from the point of greatest strain towards the supported points, in a certain ratio, without any diminution of useful strength; hence, a great saving of the material can be effected, or with the same quantity of material, a considerable accession of strength can be obtained, for this reason, that a portion of the metal may be withdrawn from those parts of the beam where the strain is least, and accumulated at or near the middle, where the straining force produces a maximum effect. Let us, therefore, inquire what will be the form of a beam when

cast in this manner; that is, when it is equally strong at every point of its length.

Now, we have already seen that

$$m\,n\,w = 212\tfrac{1}{2}\,l\,b\,d^2.$$

(*See* Case 2, Prob. I. page 6).

Therefore, supposing the length and breadth to remain the same, it is evident that d^2 varies as mnw; but w is by supposition constant for every value of mn; hence d^2 varies simply as mn. Now, mn is the rectangle of the segments into which the length of the beam is divided at the point of strain, consequently the square of the depth varies as the rectangle of the segments. By adverting to the fundamental property of the ellipsis, or the relation that subsists between the abscissæ and their corresponding ordinates, it will be seen that they follow precisely the same law; we therefore conclude, that when the length and breadth are constant, if the beam be equally strong at every point of its length, the outline of the depth must be an ellipsis.

We shall now examine if the foregoing reasoning be correct; and for this purpose let us suppose that a cast iron beam, 20 feet long and one inch broad, bears a load of 4480 lbs. at the middle of its length, while the elastic force remains unimpaired; and for the sake of greater simplicity, let the effect pro-

duced by the weight of the beam at the several points of division be always supposed to be included in the effect produced by 4480 lbs. or, in other words, let the strain produced by the load, together with the weight of the beam, be a constant quantity; then, for the greatest depth, or that at the middle where the strain is a maximum, we have

$$1 \ldots d = \sqrt{\frac{10 \times 10 \times 4480}{212\cdot5 \times 20}} = 10\cdot267 \text{ inches.}$$

Now, if we suppose half the length to be divided into ten equal parts, and calculate the depth for each point of division, we shall have the following series of depths:

$$\begin{aligned}
2 \ldots d &= 3\cdot0801 \sqrt{11} = 10\cdot216, \\
3 \ldots d &= 4\cdot1068 \sqrt{6} = 10\cdot057, \\
4 \ldots d &= 1\cdot0267 \sqrt{91} = 9\cdot793, \\
5 \ldots d &= 2\cdot0534 \sqrt{21} = 9\cdot409, \\
6 \ldots d &= 5\cdot1335 \sqrt{3} = 8\cdot891, \\
7 \ldots d &= 1\cdot0267 \times 8 = 8\cdot214, \\
8 \ldots d &= 1\cdot0267 \sqrt{51} = 7\cdot332, \\
9 \ldots d &= 1\cdot0267 \times 6 = 6\cdot160, \\
10 \ldots d &= 1\cdot0267 \sqrt{19} = 4\cdot474.
\end{aligned}$$

If these ordinates be calculated from the property of the ellipsis, they will be found to be exactly the same; hence it is evident, that when the beam is equally strong at every point of its length, the form must be elliptical.

We are hence enabled to compare the advantages or disadvantages that arise from the use of rectangular or elliptic beams.

Let it, for instance, be proposed to determine the difference of strength between a rectangular and elliptic beam, when loaded at the middle, the length, breadth, and quantity of metal being the same in both.

Since the length, breadth, and quantity of metal are the same, it is evident that the area of the vertical longitudinal sections must also be the same; hence, the greatest depth of the elliptic beam must exceed the depth of the rectangular one, to produce the same area. Now the writers on mensuration have shewn, that the area of an ellipsis is equal to the product of the two axes drawn into the decimal ·7854; therefore, the comparative depths of a rectangle and an ellipsis of the same length and area, are as 1 to $\frac{1}{\cdot 7854}$. But the strength of beams having the same length and breadth, and placed under the same circumstances, are as the squares of the depth; hence, the strength of a rectangular beam is to the strength of an elliptic one of the same length and breadth, and containing the same quantity of metal, as 1 to 1·62, or as 10 to 16·2 nearly: therefore we conclude, that when the flexure of the beam is not an object of consequence, there is a great advantage in making the form elliptical: but to return.

The six cases that we have already considered under the head of *resistance to impulsion*, apply to beams that are at rest, and operated on by a force in motion, such as the force excited by the impact of a heavy body falling freely by the force of gravity; we shall, therefore, in the next place, consider the resistance of beams that are themselves in motion, and operated on by a moving force, such as the parts of machinery, &c. Now, it is easy to shew, that the weight of a uniform rectangular beam of cast iron is expressed by $3{\cdot}2\,b\,d\,l$; where 3·2 is the weight in lbs. of a bar of that material, one inch square and twelve inches long, and $b\,d\,l$ is the area of the transverse section, drawn into the length of the beam in feet. But it is a well-known fact in mechanics, that the whole mass of the beam would acquire the same momentum only, as if half that mass were collected at the middle point (*see* TREDGOLD on Cast Iron, art. 261); therefore the above expression becomes $1{\cdot}6\,bdl$, for half the weight of the beam.

In the solution of the first case of the first problem, it is shewn that

$$l\,w = 850\,b\,d^2;$$

and in the solution of the tenth problem, the corresponding deflexion is shewn to be

$$d\,\delta = \frac{144\,l^2}{7225}.$$

Therefore, if f be the moving force and v its velocity, then, by the principles of mechanics, we have

$$f v = b d l \sqrt{\frac{6176}{85}}.$$

(*See* Tredgold on Cast Iron, art. 279).

And from this equation, slightly modified, we obtain,

7. *When the beam is uniform in breadth and depth, supported at the ends, and the moving force acts at the middle,*

$$1000 f v = 8524\, b d l.$$

8. *When the beam is parabolic, supported at the ends, and the moving force acts at the middle,*

$$1000 f v = 10782\, b d l.$$

9. *When the beam is uniform in breadth and depth, fixed on a centre of motion, with the force acting at one end, and resisted at the other,*

$$1000 f v = 8524\, b d l \sqrt{1 + R}.^*$$

10. *When the beam is parabolic, fixed on a centre of motion, with the force acting at one end, and resisted at the other,*

$$1000 f v = 8803\, b d l \sqrt{1 + R}.$$

If the arms of the beam between the centre of motion and each extremity be equal, as is generally

* In calculating the deflexion for a beam, when fixed at one end and loaded at the other (page 42), it was shewn that R is the quotient that arises from the division of the fixed part by the projecting part of the beam.

the case, the equations 9 and 10 will become respectively

$$9^a \ldots . 1000 f v = 12055 l,$$

$$10^a \ldots . 1000 f v = 12449 b d l.$$

The following examples will shew the practical application of the foregoing formulæ.

Example 10. A uniform rectangular beam of cast iron, 12 feet long and $2\frac{1}{2}$ inches broad, has a power of 3000 lbs. applied at its middle point; what must be the depth of the beam to resist that power, supposing its velocity to be 3 feet per second?

The formula is that belonging to case 7, viz.

$$1000 f v = 8524\, b d l.$$

But $b = 2\frac{1}{2}$ inches, $l = 12$ feet, $f = 3000$ lbs. and $v = 3$ feet; let these numbers be substituted in the equation for b, l, f, and v, and we obtain

$$8524 \times 2\tfrac{1}{2} \times 12 \times d = 1000 \times 3000 \times 3;$$

that is, $d = \frac{75000}{2131} = 35 \cdot 2$ inches nearly, for the depth of the beam required.

Example 11. The data remaining; what must be the depth of a parabolic beam, when placed under the same circumstances?

The formula is that belonging to Case 8, viz.

$$1000 f v = 10782 b d l.$$

Here, $b = 2\frac{1}{2}$ inches, $l = 12$ feet, $f = 3000$ lbs. and $v = 3$ feet. Let these numbers be substituted in the equation for b, l, f and v, and we have

$$10782 \times 2\tfrac{1}{2} \times 12 \times d = 1000 \times 3000 \times 3;$$

that is, $d = \frac{50000}{1797} = 27 \cdot 82$ inches nearly, for the depth of the beam required.

Example 12. A uniform rectangular beam of cast iron, 24 feet long and 34 inches deep, is fixed on a centre of motion at the middle of its length; what must be the breadth of the beam to withstand a force of 5000 lbs., moving with a velocity of 4 feet per second?

The formula is that reduced for Case 9[a], viz.

$$1000 f v = 12055\, b d l.$$

Here, $d = 34$ inches; and since the beam in this example is fixed on a centre of motion at the middle of the length, $l = 12$ feet, $f = 5000$ lbs. and $v = 4$ feet. Let these numbers be substituted for $d, l, f,$ and v, and we obtain

$$12055 \times 34 \times 12 \times b = 1000 \times 5000 \times 4;$$

that is, $b = \frac{500000}{122961} = 4 \cdot 06$ inches nearly, for the breadth sought.

Example 13. The data remaining; what must be the breadth of a parabolic beam, when placed under the same circumstances?

The formula is that reduced for Case 10[a], viz.

$$1000 f v = 12449\, b d l.$$

Now, $d = 34$ inches, $l = 12$ feet, $f = 5000$ lbs. and $v = 4$ feet; substitute these numbers in the equation, and it becomes

$$12449 \times 34 \times 12 \times b = 1000 \times 5000 \times 4;$$

that is, $b = \frac{2500000}{634899} = 3{\cdot}93$ inches nearly,

for the breadth required.

Equations similar to the above may easily be deduced for beams of the grooved and open forms, whether they be uniform in depth or cast in shape of a parabola or an ellipsis; but since the steps of derivation and reduction would be similar to what we have already done, we have thought proper to omit them.

APPENDIX:

CONTAINING

THE RULES IN WORDS AT LENGTH

FOR

CALCULATING THE MOST IMPORTANT CASES THAT HAVE BEEN INVESTIGATED IN THE FOREGOING PART OF THIS WORK;

BEING INTENDED AS

A GUIDE FOR THOSE WHO ARE NOT VERSED IN

ALGEBRAIC REDUCTIONS.

RULES

FOR

THE TRANSVERSE STRENGTH.

RECTANGULAR BEAMS.

1. *When the beam is rectangular, supported horizontally at the ends and loaded in the middle* (*see* Case 1, Prob. I. page 5).

Example 1. In the equation

$$lw = 850\, bd^2,$$

there are given, the breadth b, the depth d, and the length of bearing or distance between the supports l; to find the weight w.

Rule. Multiply 850 times the breadth in inches by the square of the depth in inches, and divide the product by the length of bearing in feet, for the weight to be supported in pounds.

Example 2. In the equation

$$lw = 850\, b\, d^2,$$

there are given, the depth d, the length of bearing l, and the load w; to find the breadth b.

Rule. Multiply the length of bearing, or distance between the supports in feet, by the given load, and

divide the product by 850 times the square of the depth in inches, for the breadth required.

Example 3. In the equation

$$l w = 850 b d^2,$$

there are given, the breadth b, the length of bearing l, and the load w; to find the depth d.

Rule. Multiply the length of bearing, or distance between the supports in feet, by the load to be supported in pounds, divide the product by 850 times the breadth in inches, and extract the square root of the quotient, for the depth required.

2. *When the beam is rectangular, supported horizontally at the ends, and loaded at some intermediate point* (*see* Case 2, Prob. I. page 6).

Example 4. In the equation

$$m n w = 212 l b d^2,$$

there are given, the breadth b, the depth d, and m and n the segments of the length of bearing; to find the load w.

Rule. Multiply 212 times the breadth in inches by the sum of the segments of the length of bearing in feet, drawn into the square of the depth in inches, and divide the product by the rectangle of the segments, for the load to be supported in pounds.

Example 5. In the equation

$$m n w = 212 l b d^2,$$

there are given, the depth d, the load w, and m and n,

the segments of the length of bearing; to find the breadth b.

Rule. Multiply the given load in pounds by the rectangle of the segments of the length of bearing in feet, and divide the product by 212 times the length, or sum of the segments, drawn into the square of the depth in inches, for the breadth required.

Example 6. In the equation

$$m n w = 212 \, l b d^2,$$

there are given, the breadth b, the load w, and m and n the segments of the length of bearing; to find the depth d.

Rule. Multiply the given load by the rectangle of the segments of the length of bearing in feet, divide the product by 212 times the sum of the segments, drawn into the breadth in inches, and extract the square root of the quotient for the depth required.

Example 7. In the equation

$$m n w = 212 \, l b d^2,$$

there are given, the breadth b, the depth d, the length of bearing or sum of the segments l, and the load w; to find m and n, the segments of the length separately.

Rule. Multiply 850 times the length of bearing in feet by the breadth drawn into the square of the depth, both in inches; divide the product by the given load, and subtract the quotient from the

square of the length in feet; then, to or from the length add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

NOTE. The formula for the third, fourth, fifth, and sixth cases of the first problem, being precisely of the same form as that corresponding to the first case, with the exception of the constant number, it seems unnecessary to repeat the rules; because, if the constant belonging to each particular case be substituted for and employed instead of 850, the rules in other respects would be precisely the same.

We therefore proceed to those cases given in the note, page 8, and

1. *When the beam is rectangular, supported horizontally at the ends, loaded in the middle, and the depth any given number of times greater than the breadth.* (*See* Case 1 of the first class, page 8).

Example 8. In the equation

$$l w x = 850 d^3,$$

there are given, the depth d, the length of bearing or distance between the supports l, and the number of times that the depth exceeds the breadth x; to find the load or weight w that the beam will sustain.

Rule. Divide eight times the cube of the depth in inches by the length of bearing in feet, drawn into the number of times that the depth exceeds

the breadth, and the quotient will give the load required in pounds.

Example 9. In the equation

$$l w x = 850 d^3,$$

there are given, the depth d, the length of bearing l, and the load w; to find x, the number of times that the depth exceeds the breadth.

Rule. Divide 850 times the cube of the depth in inches by the load in pounds, drawn into the length of bearing in feet, and the quotient will express the number of times that the depth is greater than the breadth.

Example 10. In the equation

$$l w x = 850 d^3,$$

there are given, the length of bearing l, the load w, and the number x, which expresses how often the depth is greater than the breadth; to find the depth d.

Rule. Multiply together the length of bearing in feet, the load to be supported in pounds, and the number expressing how often the depth contains the breadth; then divide the product by 850, and the cube root of the quotient will be the depth required. Let the depth thus found be divided by the number which denotes the ratio of the depth to the breadth, and the breadth itself will become known.

2. *When the beam is rectangular, supported hori-*

zontally at the ends, loaded at some intermediate point, and the ratio between the breadth and depth given. (*See* Case 2 of the first class, page 8).

Example 11. In the equation

$$m\,n\,w\,x = 212\,l\,d^3,$$

there are given, the depth d, the length l, the ratio between the breadth and depth x, and m and n the segments of the length; to find the load w.

Rule. Divide 212 times the length in feet, drawn into the cube of the depth in inches, by the rectangle of the segments of the length, drawn into the number which denotes how often the depth contains the breadth, and the quotient will be the load in pounds that the beam will bear.

Example 12. In the equation

$$m\,n\,w\,x = 212\,l\,d^3,$$

there are given, the depth d, the length of bearing l, the load w, and m and n the segments of the length; to find x, the number of times that the depth contains the breadth.

Rule. Divide 212 times the length of bearing in feet, drawn into the cube of the depth in inches, by the rectangle of the segments of the length, drawn into the load in pounds, and the quotient will be the number of times that the depth contains the breadth.

Example 13. In the equation

$$m\,n\,w\,x = 212\,l\,d^3,$$

there are given, the length l, the segments of the length m and n, the load w, and the number of times that the depth contains the breadth x; to find the depth d.

Rule. Multiply together the rectangle of the segments of the length, the load in pounds, and the number that expresses how often the depth exceeds the breadth; divide the product by 212 times the length of bearing, and extract the cube root of the quotient for the depth required.

Example 14. In the equation

$$m n w x = 212\, l d^3,$$

there are given, the depth d, the length l, the load w, and the number x which expresses how often the depth contains the breadth; to find m and n, the segments of the length, separately.

Rule. Multiply 850 times the length of bearing in feet, by the cube of the depth in inches; divide the product by the load in pounds, drawn into the number of times that the depth contains the breadth, and subtract the quotient from the square of the length in feet; then, to or from the length, add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

Note. The other cases of this class have their corresponding formula similar to that of the first case, and consequently separate rules are unneces-

sary, observing only to use the constants for those cases respectively, instead of 850, after the same manner as we stated in the preceding Note. We now proceed to consider the second class of equations, given at page 8.

SQUARE BEAMS.

1. *When the beam is square, supported horizontally at the ends, loaded in the middle, and the direction of the straining force parallel to the side.* (*See* Case 1 of the second class, page 8).

Example 15. In the equation

$$lw = 850\,s^3,$$

there are given, the length of bearing l, and the side of the square s; to find w, the load that the beam will sustain.

Rule. Divide 850 times the cube of the side in inches, by the length of bearing in feet, and the quotient will be the load required in pounds.

Example 16. In the equation

$$lw = 850\,s^3,$$

there are given, the length of bearing l, and the load w; to find s, the side of the square.

Rule. Multiply the length of bearing in feet by the load in pounds; divide the product by 850, and the cube root of the quotient is the side of the square required in inches.

2. *When the beam is square, supported horizontally*

at the ends, loaded at some intermediate point, and the direction of the straining force parallel to the side. (*See* Case 2 of the second class, page 8).

Example 17. In the equation

$$m n w = 212 l s^3,$$

there are given, the length of bearing l, the side s, and m and n, the segments of the length; to find the load w.

Rule. Divide 212 times the length of bearing in feet, drawn into the cube of the side in inches, by the rectangle of the segments of the length, and the quotient will be the load required in pounds.

Example 18. In the equation

$$m n w = 212 l s^3,$$

there are given, the length of bearing l, the load w, and m and n the segments of the length; to find s, the side of the square.

Rule. Multiply the rectangle of the segments of the length by the load in pounds; divide the product by 212 times the sum of the segments, or length of bearing in feet, and extract the cube root of the quotient for the side of the square required.

Example 19. In the equation

$$m n w = 212 l s^3,$$

there are given, the length of bearing l, the load w, and the side of the square s; to find m and n, the segments of the length separately.

Rule. Multiply 850 times the length of bearing in

feet by the cube of the side in inches; divide the product by the given load, and subtract the quotient from the square of the length in feet; then, to or from the length add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

NOTE. The remaining formulæ of this class being similar in form to the first, we think it unnecessary to draw up separate rules for their reduction; and we shall here remark, that in what follows, the rules will be given for the first and second cases of each class only; those for the first case being applicable to the third, fourth, fifth, and sixth cases, if the corresponding constants for those cases be employed.

We may further observe, that the rules which we have just given for the square beam when the direction of the straining force is parallel to the side, apply also to the case when the direction of the force coincides with the vertical diagonal, provided 600 be substituted for 850, and 150 for 212. We, therefore, pass on to the cylindrical form of beams, as treated of in Problem III. page 10.

CYLINDRICAL BEAMS.

1. *When the beam is cylindrical, supported horizontally at the ends, and loaded in the middle.* (*See* Case 1, Prob. III. page 11).

Example 20. In the equation

$$lw = 500\,d^3,$$

there are given, the length of bearing l, and the diameter d; to find w, the load that the beam will sustain.

Rule. Divide 500 times the cube of the diameter in inches by the length of bearing, or distance between the supports in feet, and the quotient will give the load required in pounds.

Example 21. In the equation

$$lw = 500\,d^3,$$

there are given, the length of bearing l, and the load w; to find d, the diameter of the cylinder.

Rule. Multiply the length of bearing in feet by the load to be supported in pounds; divide the product by 500, and the cube root of the quotient will give the diameter of the cylinder in inches.

2. *When the beam is cylindrical, supported at the ends, and loaded at some intermediate point.* (*See* Case 2, Prob. III. page 11).

Example 22. In the equation

$$mnw = 125\,l\,d^3,$$

there are given, the diameter of the cylinder d, the length of bearing l, and m and n the segments of the length; to find w, the load that the beam will sustain.

Rule. Divide 125 times the length of bearing in feet, drawn into the cube of the diameter in inches,

by the rectangle of the segments of the length, and the quotient will be the load required in pounds.

Example 23. In the equation

$$mnw = 125\,l\,d^3,$$

there are given, the length of bearing l, the load w, and m and n the segments of the length; to find d, the diameter of the cylinder.

Rule. Multiply the load in pounds by the rectangle of the segments of the length in feet; divide the product by 125 times the length, and extract the cube root of the quotient for the diameter of the cylinder.

Example 24. In the equation

$$mnw = 125\,l\,d^3,$$

there are given, the diameter of the cylinder d, the length l, and the load w; to find m and n, the segments of the length separately.

Rule. Multiply 500 times the length of bearing in feet by the cube of the diameter in inches; divide the product by the given load, and subtract the quotient from the square of the length in feet: then, to or from the length add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

For rules that apply to the remaining cases, see the preceding Notes. The following apply to hol-

low cylinders, or tubular beams, as discussed in Problem IV. page 11.

TUBULAR BEAMS.

1. *When the beam is tubular, supported horizontally at the ends, and loaded in the middle.* (*See* Equation A, Prob. IV. page 12).

Example 24. In the equation

$$l w = 500\, d^3 (1 - p^4),$$

there are given, the length of bearing l, the exterior diameter d, and the quotient of the interior diameter divided by the exterior p; to find the load w, that the beam will sustain.

power)

Rule. From unity subtract the fourth ~~part~~ of the quotient that arises, when the interior diameter is divided by the exterior; multiply the remainder by 500 times the cube of the exterior diameter in inches; then, divide the product by the length of bearing in feet, and the quotient will be the load in pounds that the beam will sustain.

Example 25. In the equation

$$l w = 500\, d^3 (1 - p^4,)$$

there are given, p the quotient that arises when the interior diameter is divided by the exterior diameter of the beam, the length of bearing l, and the load w; to find d, the exterior diameter.

Rule. Multiply the length of bearing by the given load; then divide the product by 500 times the dif-

ference between unity and the fourth power of the quotient that arises when the interior diameter is divided by the exterior, and extract the cube root of the quotient for the exterior diameter required.

2. *When the beam is tubular, supported horizontally at the ends, and loaded at some intermediate point.* (*See* Case 2, Prob. IV. page 13).

Example 26. In the equation

$$m\,n\,w = 92\,l\,d^3,$$

there are given, the exterior diameter d, the length of bearing l, and m and n the segments of the length; to find w, the load that the beam will sustain.

Rule. Divide 92 times the length of bearing in feet, drawn into the cube of the diameter in inches, by the rectangle of the segments of the length, and the quotient will be the load that the beam will sustain.

Example 27. In the equation

$$m\,n\,w = 92\,l\,d^3,$$

there are given, the length of bearing l, the load w, and m and n the segments of the length; to find d, the exterior diameter.

Rule. Multiply the rectangle of the segments of the length by the load in pounds; divide the product by 92 times the length of bearing, and extract the cube root of the quotient for the diameter required; multiply the diameter thus found by ·7166, and the product is the interior diameter.

Example 28. In the equation

$$m n w = 92\, l d^3,$$

there are given, the exterior diameter d, the length of bearing l, and the load w; to find m and n, the segments of the length separately.

Rule. Multiply 368 times the length of bearing in feet, by the cube of the diameter in inches; divide the product by the load in pounds, and subtract the quotient from the square of the length in feet; then, to or from the length, add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

NOTE. The general equation for this case is not given, but it can easily be supplied, as indeed it can for the other cases also, by merely annexing the parenthetical part of the general equation (A), Problem IV. to the several equations in Problem III.

The rules for the last case will therefore be as follows: —

Example 29. In the equation

$$m n w = 125\, l d^3 (1 - p^4),$$

there are given, the exterior diameter d, the length of bearing l, the segments of the length m and n, and also the quotient that arises when the interior diameter is divided by the exterior; to find w, the load that the beam will sustain.

Rule. From unity subtract the fourth power of the quotient that arises when the interior diameter is divided by the exterior; multiply the remainder by

125 times the length of bearing in feet, drawn into the cube of the exterior diameter in inches; then, divide the product by the rectangle of the segments of the length, and the quotient will be the load required in pounds.

Example 30. In the equation

$$m\,n\,w = 125\,l\,d^3(1-p^4),$$

there are given, the length of bearing l, the segments of the length m and n, the load w, and the quotient that arises when the interior diameter is divided by the exterior; to find d, the exterior diameter.

Rule. From unity subtract the fourth power of the interior diameter divided by the exterior; multiply the remainder by 125 times the length of bearing in feet, and reserve the product for a divisor.

Multiply the rectangle of the segments of the length in feet by the load in pounds; then, divide the product by the reserved divisor, and extract the cube root of the quotient for the diameter required.

Example 31. In the equation

$$m\,n\,w = 125\,l\,d^3(1-p^4),$$

there are given, the exterior diameter d, the length of bearing l, the load w, and the quotient of the interior diameter divided by the exterior; to find m and n, the segments of the length.

Rule. From unity subtract the fourth power of the interior diameter divided by the exterior; multiply the remainder by 500 times the length of

bearing in feet, drawn into the cube of the given diameter in inches, and divide the product by the load in pounds; subtract the quotient from the square of the length of bearing in feet; then, to or from the length, add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

For the method of applying the rules to the other cases, see the Notes.

The following rules apply to the cases in Problem V. page 14.

GROOVED BEAMS.

1. *When the beam is grooved, supported horizontally at the ends, and loaded in the middle.* (*See* Equation B, Prob. V. page 14).

Example 32. In the equation

$$lw = 850\, b\, d^2 (1 - q p^3),$$

there are given, the breadth b, the depth d, the length of bearing l, and the fractions p and q; to find w, the load that the beam will sustain.*

Rule. From unity subtract the product that arises when the quantity q is multiplied by the third power of p; then multiply the remainder by 850 times the breadth drawn into the square of the depth, both in

* For the method of finding p and q, see the solution of the fifth problem, at page 14.

inches; divide the product by the length of bearing in feet, and the quotient will give the load required in pounds.

Example 33. In the equation

$$l\,w = 850\,b\,d^2\,(1 - q\,p^3),$$

there are given, the depth d, the length of bearing l, the load w, and the fractions p and q; to find the breadth b.

Rule. Multiply the length of bearing in feet by the load in pounds, and reserve the product for a dividend; from unity subtract the product that arises when the quantity q is multiplied by the third power of p; multiply the remainder by 850 times the square of the depth in inches, and divide the reserved dividend by the product for the breadth required in inches.

Example 34. In the equation

$$l\,w = 850\,b\,d^2\,(1 - q\,p^3),$$

there are given, the breadth b, the length of bearing l, the load w, and the fractions p and q; to find the depth d.

Rule. Multiply the length of bearing in feet by the load in pounds, and reserve the product for a dividend; from unity subtract the product that arises when the quantity q is multiplied by the third power of p; multiply the remainder by 850 times the breadth in inches; then, divide the reserved dividend by the product, and the square root of the quotient will give the depth required in inches.

2. *When the beam is grooved, supported horizontally at the ends, and loaded at some intermediate point.*

NOTE. The general expression for this case is not given in the solution of the fifth problem, but it is supplied from the second case of Problem I. by simply annexing the above parenthetical expression to the right-hand side of the equation, as in the fol lowing example:—

Example 35. In the equation

$$mnw=212lbd^2(1-qp^3),$$

there are given, the breadth b, the depth d, the length of bearing l, the segments of the length m and n, and the fractions p and q; to find w, the load that the beam will sustain.

Rule. Multiply 212 times the length of bearing in feet, by the breadth drawn into the square of the depth, both in inches, and reserve the product; from unity subtract the product that arises when the quantity q is multiplied by the third power of p; then, multiply the reserved product by the remainder, and divide by the rectangle of the segments of the length, for the load required.

Example 36. In the equation

$$mnw=212\,l\,b\,d^2\,(1-qp^3),$$

there are given, the depth d, the length l, the load w, the segments of the length m and n, and the fractions p and q; to find the breadth b.

Rule. From unity subtract the product that

arises when the quantity q is multiplied by the third power of p; multiply the remainder by 212 times the length of bearing in feet, drawn into the square of the depth in inches, and reserve the product for a divisor.

Multiply the rectangle of the segments of the length of bearing in feet by the given load in pounds; then, divide the product by the reserved divisor, and the quotient will give the breadth required in inches.

Example 37. In the equation

$$m n w = 212\, l b d^2 (1 - q p^3),$$

there are given, the breadth b, the length l, the segments of the length m and n, the load w, and the fractions p and q; to find the depth d.

Rule. From unity subtract the product that arises when the quantity q is multiplied by the third power of p; multiply the remainder by 212 times the length of bearing in feet, drawn into the breadth in inches, and reserve the product for a divisor.

Multiply the rectangle of the segments of the length of bearing by the given load, divide the product by the reserved divisor, and the square root of the quotient is the depth required in inches.

Example 38. In the equation

$$m n w = 212\, l b d^2 (1 - q p^3),$$

there are given, the breadth b, the depth d, the length l, the load w, and the fractions p and q; to find m and n, the segments of the length, separately.

Rule. From unity subtract the product that arises when the quantity q is multiplied by the third power of p, and multiply the remainder by 850 times the length in feet, drawn into the breadth and square of the depth, both in inches; divide the product by the load in pounds, and subtract the quotient from the square of the length; then, to or from the length of bearing in feet, add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

The rules for the other cases of the fifth problem may be inferred from the notes. The following apply to the cases in the sixth problem, page 15.

OPEN BEAMS.

1. *When the beam is open, supported horizontally at the ends, and loaded in the middle.* (*See* Equation C, Prob. VI. page 16).

Example 39. In the equation

$$l w = 850\, b\, d^2\, (1 - p^3),$$

there are given, the breadth b, the depth d, the length of bearing l, and the fraction p, to find w, the load that the beam will sustain.*

Rule. From unity subtract the third power or

* For the method of finding p, see the solution of the sixth problem, page 15.

cube of the quantity p; multiply the remainder by 850 times the breadth drawn into the square of the depth, both in inches, and divide the product by the length of bearing in feet, for the load to be supported in pounds.

Example 40. In the equation

$$l w = 850\, b\, d^2\, (1 - p^3),$$

there are given, the depth d, the length l, the load w, and the fraction p; to find the breadth b.

Rule. From unity subtract the cube or third power of p, and multiply the remainder by 850 times the square of the depth in inches, and reserve the product for a divisor.

Multiply the rectangle of the segments of the length by the given load; then, divide the product by the reserved divisor, and the quotient will be the breadth required in inches.

Example 41. In the equation

$$l w = 850\, b\, d^2\, (1 - p^3),$$

there are given, the breadth b, the length of bearing l, the load w, and the fraction p; to find the depth d.

Rule. Multiply 850 times the breadth in inches by the difference between unity and the third power of p, and reserve the product for a divisor; multiply the length of bearing in feet by the load to be supported in pounds, and divide the product by the

reserved divisor, then, the square root of the quotient will be the depth required in inches.

2. *When the beam is open, supported horizontally at the ends, and loaded at some intermediate point.*

The general expression for this case is not given in the solution of the sixth problem: for the method of supplying it, see the preceding note.

Example 42. In the equation

$$m n w = 212\, l b d^2 (1 - p^3),$$

there are given, the breadth b, the depth d, the length l, the segments of the length m and n, and the fraction p; to find w, the load that the beam will sustain.

Rule. Multiply altogether 212 times the length of bearing in feet, the breadth and the square of the depth, both in inches, and the difference between unity and the cube, or third power of p; then, divide the product by the rectangle of the segments of the length in feet, for the load to be supported in pounds.

Example 43. In the equation

$$m n w = 212\, l b d^2 (1 - p^3),$$

there are given, the depth d, the length l, the segments of the length m and n, the load w, and the fraction p; to find the breadth b.

Rule. Multiply altogether 212 times the length of bearing in feet, the square of the depth in inches, and the difference between unity and the cube, or

third power of p, and reserve the product for a divisor; multiply the rectangle of the segments of the length by the load to be supported in pounds, and divide the product by the reserved divisor, for the breadth required in inches.

Example 44. In the equation

$$mnw = 212\,l\,b\,d^2\,(1 - p^3),$$

there are given, the breadth b, the length l, the segments of the length m and n, the load w, and the fraction p; to find the depth d.

Rule. Multiply together 212 times the length of bearing in feet, the breadth in inches, and the difference between unity and the cube, or third power of p, and reserve the product for a divisor; multiply the rectangle of the segments of the length by the load to be supported in pounds, and divide the product by the reserved divisor; then, the square root of the product will be the depth in inches.

Example 45. In the equation

$$mnw = 212\,l\,b\,d^2\,(1 - p^3),$$

there are given, the breadth b, the depth d, the length l, the load w, and the fraction p; to find m and n, the segments of the length separately.

Rule. From unity subtract the cube or third power of the quantity p; then, multiply together the remainder, the length of bearing in feet, the breadth and square of the depth, both in inches, and the constant number 850; divide the product

by the given load, and subtract the quotient from the square of the length; then, to or from the length, add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

The rules for the other cases of this problem may be inferred from the notes.

RULES

FOR THE DEFLEXION FROM CROSS STRAINS.

At page 43 we have given a rule for calculating the deflexion from cross strains; but since it is there enunciated in general terms, we have thought proper to supply a separate rule for each distinct form of the equations, involving different combinations of data, as follows:—

1. *When the beam is supported horizontally at the ends, and loaded in the middle.* (*See* Case 1, Prob. X. page 39).

Example 1. In the equation

$$d\delta = \cdot 02\, l^2,$$

there are given, the depth or diameter d, and the length of bearing l; to find the deflexion δ.

Rule. Multiply the square of the length in feet by the constant number ·02, and divide the product

by the depth or diameter in inches, and the quotient will be the deflexion required.

Example 2. In the equation

$$d\delta = \cdot 02\, l^2,$$

there are given, the deflexion δ, and the length of bearing l; to find the depth or diameter d.

Rule. Multiply the square of the length in feet by the constant number ·02, and divide the product by the deflexion, and the quotient will give the depth or diameter required.

NOTE. The same rules answer, whatever may be the shape of the beam, provided it is uniform throughout the length, if d be made the depth in the *rectangular*, *grooved*, and *open* beams, the side or the diagonal in a *square* beam, and the diameter in a *cylindrical* or *tubular* beam; and the same remark will apply to the other cases.

2. *When the beam is supported horizontally at the ends, and loaded at some intermediate point.* (*See* Case 2, Prob. X. page 40).

Example 3. In the equation

$$d\delta = \cdot 08\, m\, n,$$

there are given, the depth or diameter d, the length of bearing l, and m and n the segments of the length; to find the deflexion δ.

Rule. Multiply the rectangle of the segments of the length by the constant number ·08, and divide

the product by the depth or diameter in inches, and the quotient will be the deflexion required.

Example 4. In the equation

$$d\delta = \cdot 08\, m\, n,$$

there are given, the deflexion δ, and m and n the segments of the length; to find d, the depth or diameter.

Rule. Multiply the rectangle of the segments of the length by the constant number ·08, and divide the product by the given deflexion, and the quotient will be the depth or diameter required.

Example 5. In the equation

$$d\delta = \cdot 08\, m\, n,$$

there are given, the depth or diameter d, the deflexion δ, and the sum of the segments m and n; to find the segments separately.

Rule. From the square of the sum of the segments, subtract 50 times the depth or diameter drawn into the deflexion; then, to or from the sum of the segments, add or subtract the square root of the remainder, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

Since the form of the equation in the third case of this problem is the same as that in the first, the same rules will apply if the constant number ·025 be employed instead of ·02; but since the general equation for the fourth case changes its form, it

becomes necessary to adapt the rules to this change, as follows:—

4. *When the beam is fixed at one end, and loaded at the other.* (*See* Case 4, Prob. X. page 41).

Example 6. In the equation

$$d\delta = \cdot 08\, l^2 (1 + R \cos \phi),^*$$

there are given, the depth or diameter d, the length of projection l, the quantity R, and the angle of deflexion ϕ; to find the deflexion.

Rule. To unity add the product that arises when the quantity R is multiplied by the natural cosine of the angle of deflexion; multiply the sum by ·08 times the square of the length of projection; then, divide the product by the given depth or diameter, and the quotient will give the deflexion required.

Example 7. In the equation

$$d\delta = \cdot 08\, l^2 (1 + R \cos \phi),$$

there are given, the deflexion δ, the length of projection l, the quantity R, and ϕ the angle of deflexion; to find the depth or diameter d.

Rule. Multiply the quantity R by the natural cosine of the angle of deflexion, add unity to the product; then, multiply the sum by ·08 times the square of the length of projection, and divide the product by the given deflexion, and the quotient will give the depth required.

* For the method of finding R, see page 42.

NOTE. If the beam have the outline of the depth a parabola or an ellipsis, it is evident, that by using the proper constant numbers, the calculation can be effected by the rules which we have already delivered; it is therefore unnecessary to dwell longer on this part of the subject: we proceed then, to the *transverse stiffness* of beams, as under.

RULES

FOR THE TRANSVERSE STIFFNESS.

RECTANGULAR BEAMS.

1. *When the beam is rectangular, supported horizontally at the ends, and loaded in the middle.* (*See* Case 1, Prob. XI. page 50).

Example 1. In the equation

$$l^3 w = 42648\, b\, d^3 \mathrm{D},$$

there are given, the breadth b, the depth d, the length of bearing l, and the deflexion D; to find the load w.*

Rule. Multiply 42648 times the breadth in inches, by the cube of the depth in inches drawn into the

* D is the deflexion beyond which the beam is not to be strained, which, with the length, we shall always consider to be given.

given deflexion; divide the product by the cube of the length of bearing in feet, and the quotient will be the load in pounds that will produce the deflexion D.

Example 2. In the equation

$$l^3 w = 42648\, b\, d^3\, D,$$

there are given, the depth d, the length of bearing l, the load w, and the deflexion D; to find the breadth b.

Rule. Multiply the cube of the depth in inches, drawn into the given deflexion, by the constant number 42648, and reserve the product for a divisor; multiply the cube of the length of bearing in feet by the load in pounds that produces the proposed deflexion; then, divide the product by the reserved divisor, and the quotient will be the breadth required in inches.

Example 3. In the equation

$$l^3 w = 42648\, b\, d^3 D,$$

there are given, the breadth b, the length of bearing l, the load w, and the deflexion D; to find the depth d.

Rule. Multiply 42648 times the breadth in inches by the given deflexion, and divide the cube of the length in feet, drawn into the load in pounds, by the product; then, the cube root of the quotient will give the depth required in inches.

2. *When the beam is rectangular, supported hori-*

zontally at the ends, and loaded at some intermediate point. (*See* Case 2, Prob. XI. page 50).

Example 4. In the equation

$$m^2 n^2 w = 2665\, l\, b\, d^3\, \mathrm{D},$$

there are given, the breadth b, the depth d, the length of bearing l, the deflexion D, and m and n the segments of the length; to find the load w.

Rule. Multiply 2665 times the length drawn into the breadth, by the cube of the depth drawn into the given deflexion; then, divide the product by the square of the rectangle of the segments of the length, and the quotient will be the load in pounds that will produce the deflexion D.

Example 5. In the equation

$$m^2 n^2 w = 2665\, l\, b\, d^3\, \mathrm{D},$$

there are given, the depth d, the length of bearing l, the load w, the deflexion D, and m and n the segments of the length; to find the breadth b.

Rule. Multiply 2665 times the length of bearing by the cube of the depth drawn into the given deflexion, and reserve the product for a divisor; then, multiply the square of the rectangle of the segments by the given load, and divide the product by the reserved divisor for the breadth required.

Example 6. In the equation

$$m^2 n^2 w = 2665\, l\, b\, d^3\, \mathrm{D},$$

there are given, the breadth b, the length of bearing l, the load w, the deflexion D, and m and n the segments of the length; to find the depth d.

Rule. Multiply 2665 times the length of bearing by the breadth drawn into the given deflexion, and divide the square of the rectangle of the segments of the length, drawn into the load, by the product; then, the cube root of the quotient will give the depth required.

Example 7. In the equation

$$m^2 n^2 w = 2665\, l\, b\, d^3\, \mathrm{D},$$

there are given, the breadth b, the depth d, the length of bearing l, the load w, and the deflexion D; to find m and n, the segments of the length.

Rule. Multiply 42648 times the length drawn into the breadth, by the cube of the depth drawn into the deflexion, and divide the product by the given load; then, from the square of the length subtract the square root of the quotient, and again extract the square root of the remainder; then, to or from the given length add or subtract this last-found square root, and half the sum, or half the difference, will give the greater or lesser segment accordingly.

NOTE. The rules that we have delivered for the three examples of the first case, answer also for the several examples of the third case, page 51, observing to employ the constant number 68236 instead of 42648; and by adapting the last form of deflexion in the table, page 43, the constant for the fourth case, page 52, becomes 1333 to be employed instead of 42648: and, moreover, if for $b\,d^3$, in the rectangular beam, there be substituted s^4 in the

square, and d^4 in the cylinder, the several equations assume a similar form, with the exception of the constant numbers, and consequently, by employing the respective constants instead of 42648 and 2665, the rules which we have delivered for the rectangular beam apply equally to the square and cylinder, observing that $b\,d^3$ is equivalent to s^4 or d^4. We therefore pass over these forms, and proceed to consider the general expressions for the

TUBULAR BEAM.

1. *When the beam is tubular, supported horizontally at the ends, and loaded in the middle.* (*See* Prob. XIV. Case 1, Class (E), page 56).

Example 8. In the equation,

$$l^3 w = 25087\, d^4 \mathrm{D}\, (1 - p^4),$$

there are given, the diameter d, the length of bearing l, the fraction p, and the deflexion D; to find the load w.*

Rule. From unity subtract the fourth power of the fraction p; multiply the remainder by 25087 times the deflexion, drawn into the fourth power of the diameter, and divide the product by the cube of the length of bearing for the load required.

* The length of bearing l, the deflexion D, and the fraction p, are always supposed to be given: for the method of finding p, see the solution of Prob. IV. page 14, and the solution of Prob. XIV. page 56.

Example 9. In the equation

$$l^3 w = 25087\, d^4\, D\, (1 - p^4),$$

there are given, the length of bearing l, the deflexion D, the fraction p, and the load w; to find the diameter d.

Rule. From unity subtract the fourth power of the fraction p; multiply the remainder by 25087 times the deflexion, and reserve the product for a divisor; then, multiply the cube of the length of bearing by the given load, divide the product by the reserved divisor, and extract the fourth root of the quotient for the diameter required.

2. *When the beam is tubular, supported horizontally at the ends, and loaded at some intermediate point.* (*See* Prob. XIV. Case 2, Class (E), page 56).

Example 10. In the equation

$$m^2 n^2 w = 1568\, l\, d^4\, D\, (1 - p^4),$$

there are given, the diameter d, the length of bearing l, the deflexion D, the fraction p, and m and n the segments of the length; to find the load w capable of producing the deflexion D.

Rule. From unity subtract the fourth power of the fraction p, and multiply the remainder successively by the length of bearing, the deflexion, the fourth power of the diameter, and the constant number 1568; then, divide the product by the square of the rectangle of the segments of the length, and the quotient will be the load required.

Example 11. In the equation

$$m^2n^2w = 1568\, l\, d^4\, \mathrm{D}\, (1 - p^4),$$

there are given, the length of bearing l, the deflexion D, the fraction p, the load w, and m and n the segments of the length; to find the diameter d.

Rule. From unity subtract the fourth power of the fraction p; multiply the remainder by 1568 times the length, drawn into the deflexion, and reserve the product for a divisor; then, multiply the square of the rectangle of the segments of the length by the given load, divide the product by the reserved divisor, and extract the fourth root of the quotient for the diameter required.

Example 12. In the equation

$$m^2n^2w = 1568\, l\, d^4\, \mathrm{D}\, (1 - p^4),$$

there are given, the diameter d, the length of bearing l, the deflexion D, the fraction p, and the load w; to find m and n, the segments of the length.

Rule. Multiply 25087 times the length by the fourth power of the diameter drawn into the deflexion, and again by the difference betwixt unity and the fourth power of the fraction p, and divide the product by the given load; then, from the square of the length subtract the square root of the quotient, and again extract the square root of the remainder; then to or from the given length add or subtract this last found square root, and half the sum, or half the

difference, will give the greater or lesser segment accordingly.

For the method of applying the rules to the two remaining cases, see the Note to the rectangular beam preceding; and next for the

GROOVED BEAM.

1. *When the beam is grooved, supported horizontally at the ends, and loaded in the middle.* (*See* Prob. XV. Case 1, Class (F), page 57).

Example 13. In the equation

$$l^3 w = 42648\, b\, d^3\, D\, (1 - q\, p^3),$$

there are given, the breadth b, the depth d, the length l, the deflexion D, and the fractions p and q; to find the load w.*

Rule. From unity subtract the product that arises when the quantity q is multiplied by the cube of p; then, multiply the remainder by the breadth, the cube of the depth, the deflexion, and the constant number 42648, and divide the product by the cube of the length, for the load required.

Example 14. In the equation

$$l^3 w = 42648\, b\, d^3\, D\, (1 - q\, p^3),$$

there are given, the depth d, the length of bearing l,

* For the method of finding the fractions p and q, see the solution to Prob. V. page 14; which, with the deflexion and the length of bearing, are always supposed to be given.

the deflexion D, the load w, and the fractions p and q; to find the breadth b.

Rule. From unity subtract the product that arises when the quantity q is multiplied by the cube of p; then, multiply the remainder by the cube of the depth, the deflexion, and the constant number 42648, and reserve the product for a divisor; multiply the cube of the length of bearing by the given load, and divide the product by the reserved divisor, and the quotient will be the breadth required.

Example 15. In the equation

$$l^3 w = 42648\, b\, d^3\, \mathrm{D}\,(1 - q p^3),$$

there are given, the breadth b, the length l, the deflexion D, the load w, and the fractions p and q; to find the depth d.

Rule. From unity subtract the product that arises when the quantity q is multiplied by the cube of p; then, multiply the remainder by the breadth, the deflexion, and the constant number 42648, and reserve the product for a divisor; multiply the cube of the length of bearing by the given load, divide the product by the reserved divisor, and extract the cube root of the quotient for the depth required.

2. *When the beam is grooved, supported horizontally at the ends, and loaded at some intermediate point.* (*See* Prob. XV. Case 2, Class (F), page 57).

Example 16. In the equation

$$m^2n^2w = 2665\, l\, b\, d^3\, D\, (1 - q\, p^3),$$

there are given, the breadth b, the depth d, the length l, the deflexion D, fractions p and q, and segments of the length m and n; to find the load w.

Rule. Multiply the difference betwixt unity and the product that arises when the quantity q is multiplied by the cube of p, successively, by the breadth, the cube of the depth, the length of bearing, the deflexion, and the constant number 2665; then, divide the product by the square of the rectangle of the segments of the length, and the quotient will give the load required.

Example 17. In the equation

$$m^2n^2w = 2665\, l\, b\, d^3\, D\, (1 - q\, p^3),$$

there are given, the depth d, the length of bearing l, the deflexion D, the load w, the fractions p and q, and the segments of the length m and n; to find the breadth b.

Rule. Multiply 2665 times the length of bearing, drawn into the cube of the depth, by the deflexion, and again by the difference between unity and the product of q, drawn into the cube of p, and reserve the product for a divisor; multiply the square of the rectangle of the segments of the length by the given load, and divide the product by the reserved divisor, and the quotient will be the breadth required.

Example 18. In the equation

$$m^2n^2w = 2665\, l\, b\, d^3\, D\, (1 - q\, p^3),$$

there are given, the breadth b, the length of bearing l, the deflexion D, the fractions p and q, the load w, and m and n the segments of the length; to find the depth d.

Rule. Multiply 2665 times the length of bearing by the breadth, the deflexion, and the difference between unity and the quantity q multiplied by the cube of p, and reserve the product for a divisor; multiply the square of the rectangle of the segments of the length by the given load, divide the product by the reserved divisor, and the cube root of the quotient will be the depth required.

Example 19. In the equation

$$m^2 n^2 w = 2665\, l\, b\, d^3\, \mathrm{D}\, (1 - q p^3),$$

there are given, the breadth b, the depth d, the length of bearing l, the deflexion D, the load w, and the fractions p and q; to find m and n the segments of the length.

Rule. Multiply 42648 times the length drawn into the breadth, by the cube of the depth drawn into the deflexion, and again, by the difference between unity and product of q by the cube of p; divide the product by the given load; then, from the square of the length subtract the square root of the quotient, and again extract the square root of the remainder; then, to or from the given length add or subtract the square root last found, and half the

sum, or half the difference, will give the greater or lesser segment accordingly.

NOTE. The forms for the open beam, Problem XVI. Class (G) being precisely the same as those for the grooved beam in this problem, with the exception of the parenthetical expression, it seems unnecessary to give separate rules for the calculation; and the more especially as the fractional number is always supposed to be given.

The formulæ for the principal cases of *torsion* being the same as those for the corresponding cases of *transverse strength*, the rules which apply to the several cases of the one are equally applicable to the corresponding cases of the other, provided that the letter r be substituted for l in each of them: for this reason we refrain from giving the rules for the *resistance to torsion;* and we may further observe, that the useful practical cases of *compression* and *tension* being expressed by equations of such a simple form, it seems needless to give rules in words for their solution, we therefore omit them.

THE END.

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